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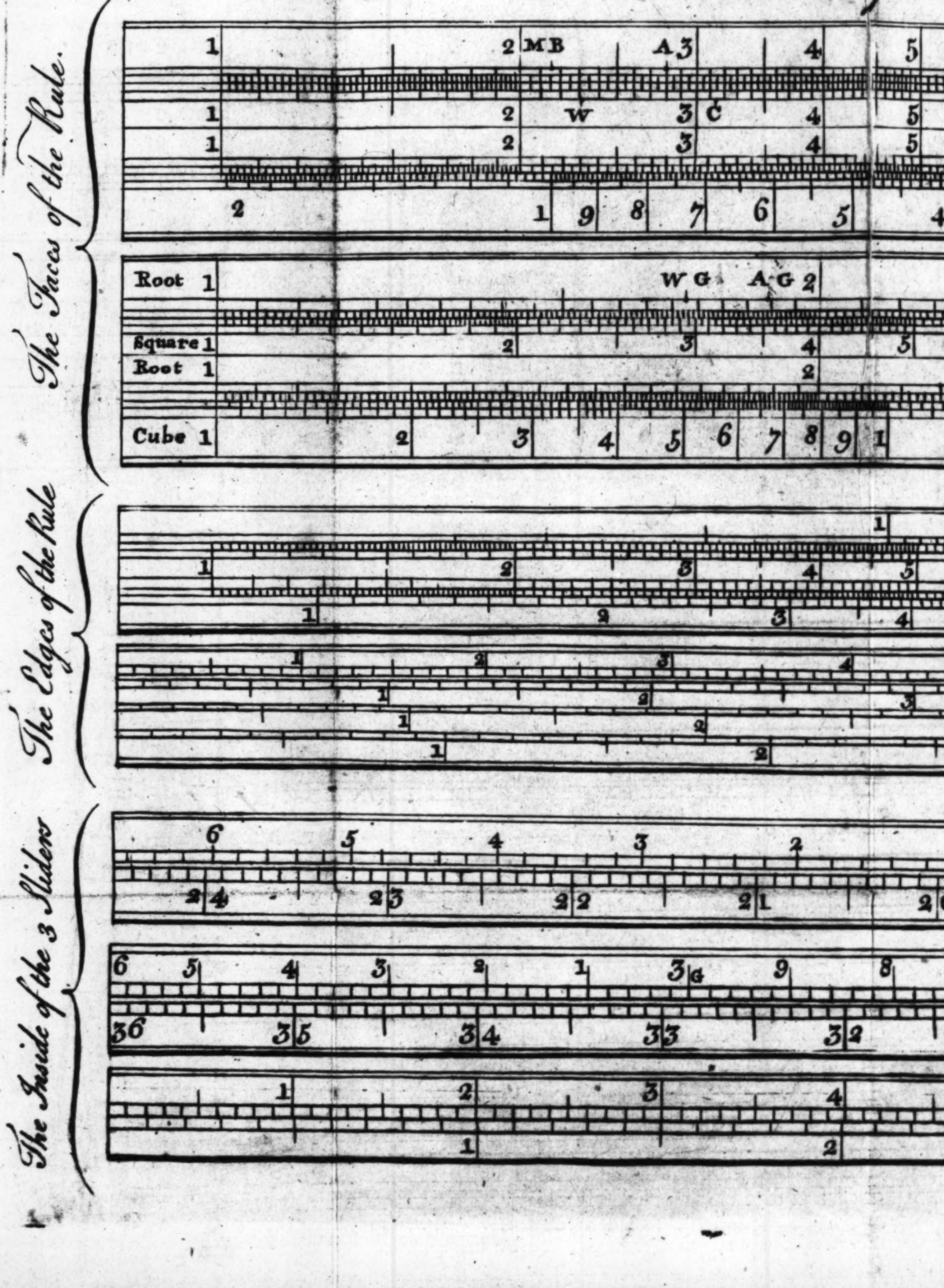
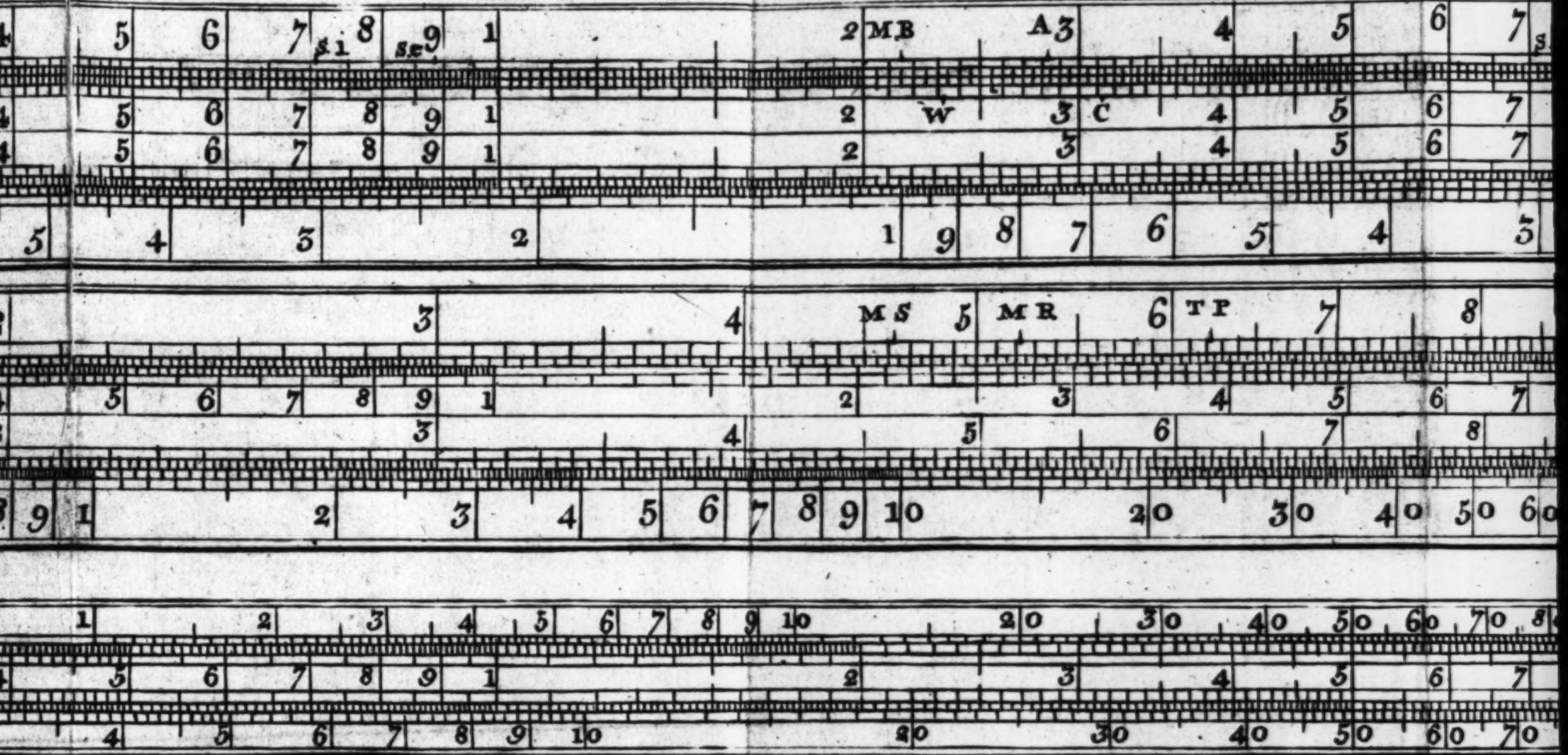

The Faces of the Rule.
The Edges of the Rule
The Inside of the 3 Sliders
MB
A
W
C
Root
WG
AG
Square
Cube

# of the Several Parts of the Sliding Rule.



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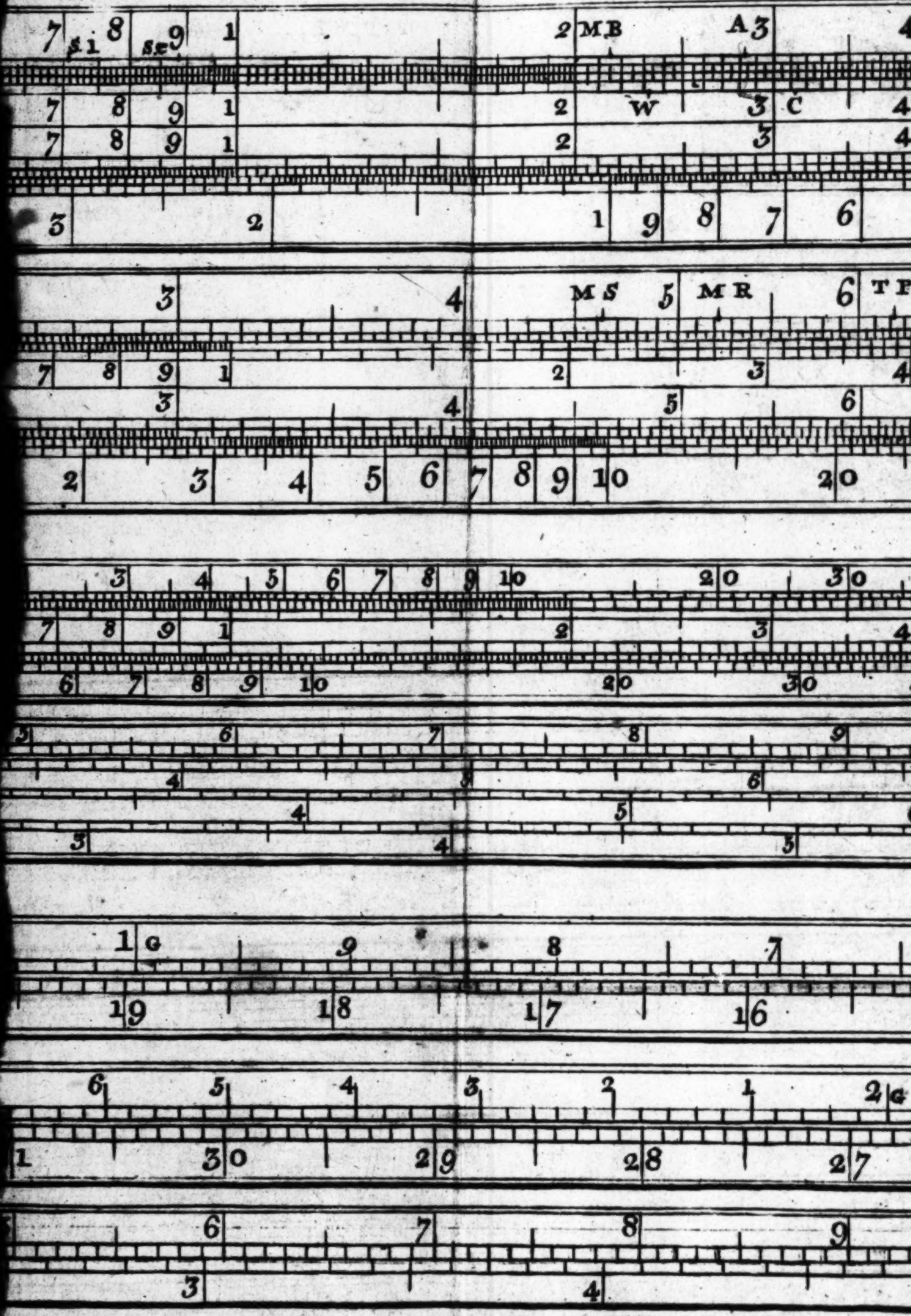
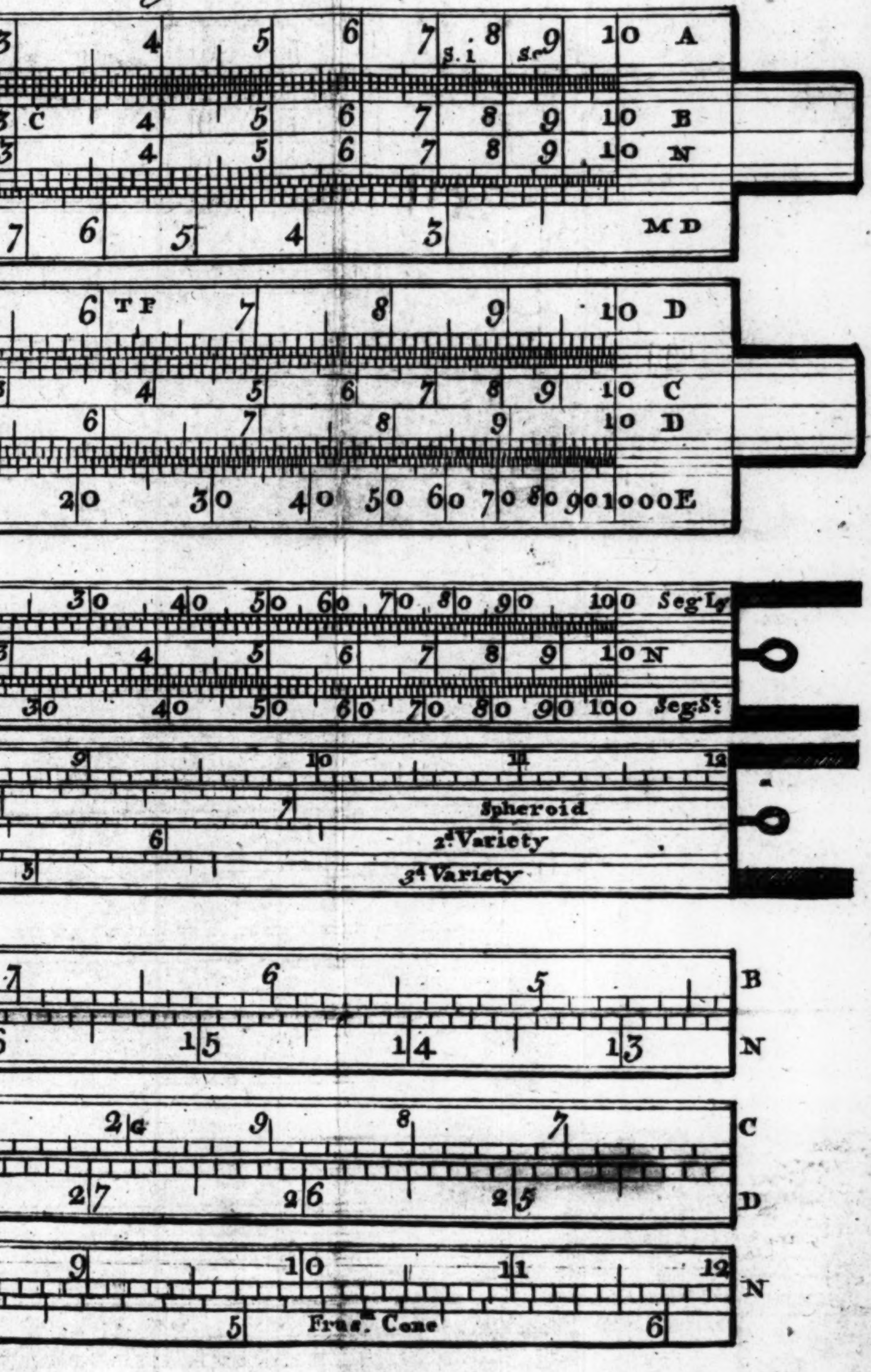


Plate I. *facing the Title*

Sliding Rule.



STEREOMETRY:

Or, The ART of

GAUGING

Made eaſy, by the Help of a

SLIDING-RULE:

Which ſhews the Area's of Circles in Gallons, and the Square and Cube-Root of any Number under 100000, by Inſpection: And is alſo very uſeful in *Arithmetic* and *Geometry*, particularly in the Meaſuring of *Superficies* and *Solids*.

WITH AN

Appendix of CONIC SECTIONS:
Shewing the NATURE, PROPERTIES, and CONTENTS of ſeveral SOLIDS, frequently mentioned in Books of GAUGING.

By THO. EVERARD, Eſq;

The ELEVENTH EDITION, Carefully Corrected.

To which are added, New EXCISE-TABLES:
Alſo the new Method of Gauging STILLS, BACKS, *&c.* with ſhort and Plain Rules for Gauging of Malt; and the Line upon the RULE called *M. D.* or the *Malt-Depth*, explained.

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WARD, and W. JOHNSTON. MDCCL.

1607/4701.

BRITISH MUSEUM

TO THE

RIGHT HONOURABLE

SIDNEY

LORD GODOLPHIN,

LATE

Lord HIGH-TREASURER

OF

ENGLAND;

This TRACT is Humbly

DEDICATED and PRESENTED

BY THE

AUTHOR.

PREFACE.

THE *Inſtrument* deſcribed in this *Tract*, (though uſeful to all Perſons concerned in the Meaſuring of *Superficies* and *Solids*) was principally deſigned for the Aſſiſtance of the *Officers* of the *Exciſe*; for whom it is now found ſo uſeful and neceſſary, that no *Gauger* is thought to be well prepared for his Buſineſs without it: But the many Thouſands that have been ſold, and the ſeveral Editions of this *Tract*, are ſo good Evidence of its Utility, that it is as needleſs as unbecoming, for me to ſay any thing in its Commendation.

PREFACE.

If the Reader finds this *Edition* ſomewhat different from ſome of the former, it is preſumed he will have no Cauſe to complain of the *Alterations*; for beſides, that this is more *Portable* and more *Correct*, here is alſo added [*Sect.* IX.] ſome ſhort and plain Rules for the *Gauging of Malt*; with a particular Account of the *Dimenſions* and *Content* of the *Standard-Buſhel*, and how the ſame were found and aſcertained; alſo a *Table* ſhewing the *Diameter* and *Depth* of any legal *Buſhel*, *Half-Buſhel*, *Peck*, or *Gallon*, with *Directions* for making of any of theſe *Meaſures*, and for Rectifying ſuch as are already made. Alſo,

New Tables of Exciſe agreeing to the Act of Parliament made in the Year 1710, for laying additional Duties upon all exciſeable Liquors, advancing the Exciſe three Pence *per Barrel* ſtrong, and one Penny *per Barrel* ſmall *Beer*, calculated for the *common Brewery* both in Town and Country; likewiſe two *Caſh-Tables* for Victuallers both for *ſtrong* and *ſmall Beer*, which were never in any of the former Editions. And that this *Book* and *Sliding-Rule* may be compleat Companions for the Officers of the Exciſe, *&c.* I have explained the *Gauge-Point*, and the Lines marked M D, or Line called the *Malt-Depth*, ſhewing its great Uſe in caſting up *Malt-Gauges.*

And here further let me acquaint the Reader, that the Line E, is not only uſeful to extract the *Cube-Root*, and to find the Solidity of a Sphere, as mentioned in Pages 44, and 70, but is alſo more particularly uſeful to all concerned in *Stereometry*, for calculating *Diagonal Lines* to any *Caſk* whatſoever, as I ſhall here ſhew, *viz.*

As the known Length of any *Diagonal Line* on the Line D, in Inches,
Is to the Content of the ſame Caſk on the Line E, in Gallons:
So is the Content of any other Caſk in Gallons, on the Line E,
To the *Diagonal* of that Caſk in Inches on the Line D.

Example. Admit a Caſk in the Form of a *Spheroid*,

whoſe { Head—22, Bung—25, Length 30. } Inches.
Diagonal 27.8
Content 48.3 Ale-Gallons.

OPERATION.

Set 27.8 on D, to 48.3 on E, and as the Rule now ſtands, it is a Table: So againſt any Number of Gallons on the Line E, you have given on the Line D, the Length of the *Diagonal* in Inches and Decimal Parts. Thus,

Content	10.	20.	30.	40.
Diagonal	16.44	20.8	23.8	26.25

For WINE.

Let the Dimenſions of a Spheroidical Caſk be as above, the Content in Wine-Gallons will be 58.97; then for the *Diagonal*, ſet 27.8 on D, to the Content 58.97 on E, and as the Rule now ſtands, you have the *Wine-Diagonal* for a *Spheroidical Caſk*. Thus,

Content	10.	20.	30.	40.
Diagonal	15.32	19.32	22.1	24.36

And ſo on *ad infinitum*.

Thus far the Author; but as ſeveral Editions have been printed ſince that time, and the preſent Proprietors being deſirous that this Edition ſhould, as far as poſſible, appear perfect, they have requeſted me to reviſe the following Sheets, and to add or alter what I thought was moſt neceſſary, which I have accordingly done, and that not *Tranſiently*, but with Care, have wrought *every Operation* over anew, both by the *Pen* and *Sliding-Rule*; ſo that I hope very few (if any) Errors have eſcaped.

And in order to render it a complete Treatiſe of Gauging independent of any other Work, I have added a conciſe Introduction to Geometry, which was wanting in the former Editions; and conſequently the young Gauger obliged to have Recourſe to ſome other Author, in order to have a proper Idea of Geometrical Figures, and the Method of conſtructing them, before he could proceed in the Practice of Gauging. I have alſo explained the Method of taking the Dimenſions of large Stills and Backs, in a very accurate Manner, whereby their Contents may be found to a much greater Degree of Exactneſs than by the Methods generally made uſe of. And as there was no Cut of the Inſtrument deſcribed in this Treatiſe, I have added a Delineation of it, as it is now improved, by which means the Young Gauger will have a proper Idea of it, and at the ſame time underſtand the Precepts with far leſs Difficulty.

J. Barrow.

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THE INTRODUCTION TO DECIMAL ARITHMETIC AND GEOMETRY.

WHOEVER will rightly underſtand the Art of *Gauging*, or the Uſe of the *Inſtruments* here deſcribed, ought in ſome meaſure to be acquainted with the art of Arithmetic; at leaſt, with *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*, both in *Whole Numbers* and *Decimal Fractions*.

And as moſt of the *Problems* in this Tract are reſolved by Help of the *Line of Numbers*, the ſeveral Uſes of which *Line* are reduced more eaſy by the Knowledge of *Decimal Arithmetic*; I ſhall, in the firſt Place, endeavour to ſhew what a *Decimal Fraction* is, and alſo give ſome Rules and Examples, by which any Perſon who does not at all underſtand the *Vulgar Arithmetic*, may, in a few Hours, thoroughly underſtand this.

A *Decimal Fraction* is that, which by prefixing a Point or a Prick towards the left Hand, its Value is decreaſed from ſo many Units, to ſo many tenth Parts of an Unit; and if a Point and a Cypher, or Digit be prefixed, it will then be ſo many hundred Parts; and if a Point and two Cyphers or Digits be prefixed, its Value is decreaſed to be but ſo many thouſand Parts; As, if you would prefix before the Figure 2, a Point thus [.2] 'tis then decreaſed from 2 Units, to 2 tenth Parts of an Unit. For,

As in *Whole Numbers* the Value or Denomination of Places *increaſes* by Tens from the Unit's Place towards the *left* Hand; ſo in *Decimal Fractions*, the Value or Denomination of Places *decreaſes* by Tens from the Unit's Place towards the *right* Hand; as in the following Scheme;

In which I have ſet an Unit in its due Place, and all the Figures towards the left Hand (being Whole Numbers) do *increaſe* by Tens; for the Figure 2, next the Unit's Place, is 2 Tens (or 20,) the next is 3 Hundreds, and the next 4 Thouſands, *&c.*

Hundreds of Thouſands.	*Tens of Thouſands.*	*Thouſands.*	*Hundreds.*	*Tens-*	*Units.*	*Tenth Parts.*	*Hundred Parts.*	*Thouſand Parts.*	*Ten Thouſand Parts.*	*Hundred Thouſand Parts.*
6	5	4	3	2	1	.2	.3	.4	.5	.6

But all the Figures from the Unit'sPlace towards the right Hand (being *Decimal Fractions*) decreaſe by Tens; ſo the Figure 2, towards the right Hand is 2 tenth Parts, the next is 3 hundred Parts, and the next 4 thouſand Parts of an Unit; each Place towards the right Hand being ten times leſs.

In

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In *Decimal Arithmetic*, we always imagine (and it would be very commodious, if it were really ſo) that all intire Units, Integers, and Things, are divided into ten equal Parts, and each of theſe Parts ſo divided, we call *Primes*; we alſo divide each of theſe *Primes* into other ten equal Parts, and every of theſe Diviſions we call *Seconds*; and each of theſe being divided into ten other equal Parts, may be called *Thirds*, (and as one *Prime* is $\frac{1}{10}$ of an Unit, ſo one Second is $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$ of an Unit; and one Third is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{1000}$ Part of an Unit;) ſo by dividing the former, and ſubdividing theſe latter, we may run on *ad infinitum*.

Let a Pound Sterling be given to be decimally divided.

According to the Notion premiſed, the firſt Diviſion muſt be *Primes*, the next Diviſion *Seconds*, the next *Thirds*, &c.

So if one Pound Sterling (being 20 *s.*) be divided into ten equal Parts, one of theſe Parts is one *Prime*, and its Value is 2 *s.* and will ſtand thus (.1) three *Primes*, or three Tenths of a Pound will ſtand thus (.3) and its Value is 6 *s.* Again, one *Prime*, or .1 being divided into ten equal Parts, each of thoſe Parts will be one Second, or one hundredth Part of a Pound, and is thus expreſs'd (.01.) and its Value will be 2 *d.* 1 *f.* and $\frac{6}{10}$ of a Farthing; and if .01 be divided into ten *other* equal Parts, each of thoſe Parts will be *Thirds*, or one thouſandth part of a Pound, and will ſtand thus (.001,) and its Value will be $\frac{96}{100}$ of a Farthing.

The like may be underſtood of one Pound *Troy* or *Averdupois*, one Foot, one Gallon, or any other Integer, or Thing whatſoever.

A *Decimal Fraction*, whether it ſtand alone or be join'd with Whole Numbers, is diſtinguiſhed from a Whole Number by a Point or Prick before it, towards the left Hand, as in theſe Examples:

.4752 4.752 47.52 475.2

In *Decimal* Fractions, the Numerators only are ſet down, the Denominators being known by the Number of Places in the Numerator: For if the Numerator conſiſt of but one Place, the Denominator is 10; if of 2 Places, the Denominator is 100, *&c.*

EXAMPLE.

The Denominator of	.2	is	10
	.25		100
	.051		1000
	.0752		10000

As Cyphers before a Whole Number have no Value, ſo Cyphers before a Decimal Fraction are of no Signification; and therefore will not increaſe the Fraction, nor alter the Denominator: For .2 is two tenth Parts, and .20 is no more.

Again, as Cyphers after a Whole Number *increaſe* that Number; ſo Cyphers before a Decimal Fraction, *diminiſh* the Value thereof; as in theſe Examples.

.5	is	5	*Tenth*	Parts.
.05			*Hundred*	
.005			*Thouſand*	
.0005			*Ten thouſand*	

Here every Cypher added, removes the Fraction further from Unity, making it ten times leſs than before.

Addition and Subtraction of Decimal Fractions.

1. AS for the Operation of Addition and Subtraction in *Decimals*, it is the very ſame with the *Vulgar*; there muſt only care be had of placing Units under Units, and Fractions under Fractions, in their proper Ranks and Files.

For Example.

To ———— 376.435
Add ———— 54.22

Sum is ——— 430.655

Again,

To ———— 26.51
Add ———— 527.342

Sum is ——— 553.852

Subtraction.

From	430.645
Subtract	54.210
Remains	376.435

Or thus:

From	553.852
Subtract	527.342
Remains	26.510

Multiplication of Decimals.

1. IN *Multiplication* both of Decimal Fractions and Mixt Numbers, there are ever two Numbers given to find a Third unknown.

One of the Numbers given (and 'tis no matter which) is called the *Multiplicator*, and the other the *Multiplicand*, and the Number ſought is called the *Product*; and this doth ever contain one of the Numbers given, as many times as the other given Number contains Unity.

Multiplication of *Decimals* is performed after the *ſame manner* as in *whole Numbers*; for the Numbers being ſet down one under another, we proceed to *Multiplication* as if they were all whole Numbers; only it muſt be obſerved (when the Operation is finiſhed) how many Places of *Decimal* Fractions are contained in the *Multiplcator* and *Multiplicand*; for ſo many muſt be in the *Product*, as in theſe Examples.

Multiplicand ———— 35.25
Multiplicator ———— 7.24

14100
7050
24675

Product ———— ———— 255.2100

Or thus:

Multiplicand ———— 6.75
Multiplicator ———— .25

3375
1350

Product ———— ———— 1.6875

2. When it happens (as it often does) that there are not ſo many *Figures* in the *Product*, as there are *Fractions* in the two Numbers to be multiply'd, you are then to make *Cyphers* before the *Product*, till the Number of Places be equal; as in theſe Examples:

.04	.75	.85
.6	.05	.005
.024	.0375	.00425

3. As *Whole Numbers* are increaſed by Multiplication, ſo *Decimal Fractions* are made leſs by it; for the *Product* is remov'd farther from Unity than either of the Fractions given to be multiply'd; as appears by the laſt Examples.

4. When a *Decimal* Fraction or Mixt Number is to be multiply'd by an Unit with Cyphers (as 10, 100, 1000, *&c.*) you need only remove the Point ſo many Places towards the right Hand, as there as Cyphers with the Unit: Thus, if .7562 be multiply'd

$$\text{by}\left\{\begin{array}{l}10\\100\\1000\\10000\end{array}\right\}\begin{array}{c}\text{The Product}\\\text{will be}\end{array}\left\{\begin{array}{l}7.562\\75.62\\756.2\\7562.\end{array}\right\}$$

Diviſion of Decimals.

1. IN Diviſion there are Two Numbers given, to find a Third unknown.

One of the Numbers given is called the *Diviſor*, the other the *Dividend*, and the Number ſought is called the *Quotient*, and the Quotient always contains Unity as many Times as the Diviſor is contained in the Dividend.

Diviſion of Decimal Fractions, or Mixt Numbers, is performed after the ſame manner as in Whole Numbers; all the Difficulty is, to know the Denomination of Places in the Quotient, and they are eaſily diſcovered by this one General Rule.

The firſt Figure in the *Quotient* muſt be of the ſame Denomination with that Figure in the Dividend, which at the firſt Demand ſtands exactly over the Place of *Unity* in the Diviſor. I ſhall make this plain by ſome Examples

EX-

EXAMPLE.

Let it be required to divide 32 *by* 51.2:

In this Example, the Diviſor (51.2) is greater than (32) the Dividend; therefore in this and the like Caſes you muſt place a competent Number of Cyphers behind the Dividend, which (if it be a Whole Number) the Cyphers ſo placed, muſt be diſtinguiſhed from it as Fractions; as, here I put 4 Cyphers behind 32; and placing the Numbers in this Manner,

(51.2) 32.0000(

I proceed to the Diviſion (as if all were Whole Numbers;) and firſt aſk, how many times the Diviſor is contained in (320) the three firſt Figures of the Dividend? And ſeeing in this Example it is not contained in the three firſt Figures, it will therefore extend to the fourth Place, over which I put a Prick, to ſhew that I muſt begin to ſubtract at that Place. I then aſk, how many times 5 in 32? I find 6; I place 6 in the Quotient, and multiply the Diviſor by 6; ſubtracting the Product out of the Dividend, beginning my Subtraction at the Place over which I put the Prick: Thus, 6 times 2 is 12, 12 from 0 I cannot; I borrow 2, and ſay 12, from 20 and there remains 8, which I place under the Cypher: Again, 6 times 1 is 6, and 2 I borrowed, is 8; 8 from 10, and there reſts 2, which I place under the next Cypher before 8, as in the Example: Then 6 times 5 is 30, and 1 I borrowed, is 31; 31 from 32,

51.2) 32.0000 (6
128

and there reſts 1, which I ſet down under 2: Then for my new Dividend I bring down the next Cypher, and ſet it behind the Remainder; and the Example will ſtand thus:

51.2) 32.0000 (6
1280

Then proceeding in my Diviſion, I aſk, how many times 5 in 12; I find 2, which I place in the Quotient, and ſay, 2 times 2 is 4; 4 from 10, and there remains 6, which I ſet under 0: Again, 2 times 1 is 2, and 1 I borrowed, is 3; 3 from 8, and there reſts 5; which I place under 8; then 2 times 5 is 10; 10 from 12 and there reſts 2; which I place under 2; and to this Remainder I bring down another Cypher to make my new Dividend; ſo will the Example ſtand thus:

51.2) 32.0000 (62
1280
256

51) 32.0000 (62
1280
2560

This done, I enquire how many times 5 in 25, and find 5; which I place in the Quotient, and ſay, 5 times 2 is 10; 10 from 10, and there reſts 0; then 5 times 1 is 5, and 1 I borrowed is 6, 6 from 6 and there reſts 0. Laſtly 5 times 5 is 25, 25 from 25, and there remains 0: and thus my Diviſion is finiſhed, and the Example will ſtand thus:

51.2) 32.0000 (.625
1280
2560
0000

Now, by the general Rule above given, 'tis plain, that the firſt Figure in the Quotient muſt be Tenths, and therefore the Quotient is all Decimal Parts; For the Diviſor being placed under the firſt Figures of the Dividend, this Example will ſtand thus:

32.0000 (.625
51.2

By which it appears, that the Cypher in the Dividend, which ſtands over the Unit's Place in the Diviſor, is the place of Tenths.

See more Examples.

Diviſor.	Dividend.	Quotient.
51.2)	32.00000	(.625
51.2)	32.00000	(.625
7.5)	.375	(.05

2. In Diviſon of *Whole* Numbers or *Mixt*, if there happen to be a Remainder, you may bring down more *Cyphers*, and by containing your *Diviſion*, carry on the *Quotient* to as many Places of *Fractions* as you pleaſe; four or five will be very near the truth; but two or three will be ſufficient in moſt caſes.

EXAMPLE.

```
6.26) 25.800000 (4.1214
        760
       1340
         880
         2540
           36
```

3. When any Number (either Decimal or Mixt) is to be divided by an Unit with Cyphers, as 10, 100, 1000, &c. you need only remove the Prick in the Dividend ſo many Places towards the left Hand, as there are Cyphers with the Unit, ſupplying the vacant Places (if any be) with Cyphers.

Thus, 756.2 divided by 10, is 75.62 ; divided by 100, it will be 7.562 ; by 1000, .7562 ; and if by 10000, the Quotient will be .07562.

4. *Diviſion* is proved by *Multiplication*, and *Multiplication* by *Diviſion*.

To prove Diviſion, multiply the Quotient by the Diviſor, and the Product is the Dividend : Thus, .625 (the Quotient in the firſt Example of Diviſion) being multiply'd by 51.2 (the Diviſor) the Product is 32.0000 (the Dividend.)

To prove Multiplication, divide the Product by either of the Numbers given to be multiply'd, and the Quotient will be the other Number given.

Thus, if the laſt Product, (*viz.*) 32.0000 be divided by 51.2, the Quotient is .625 ; or if 32.0000 be divided by .625, the Quotient we be 51.2.

And hence 'tis evident, that Diviſion is alſo proved by Diviſion : For if any Quotient be made a Diviſor, it will quote the firſt Diviſon. *Example.*

1.8) 28.8 (16.
108
000

16.) 28.8 (1.8
128
000

5. Now foraſmuch as Multiplication is eaſier than Diviſion, I ſhall here ſhew, that what is perform'd by a Diviſor, may alſo be perform'd by a Multiplicator, and how ſuch Multiplicator may be found: Suppoſe I were to divide 900 by 25, the Quotient will be 36.

Now let it be requir'd to multiply 900 by a certain Number that ſhall produce 36.

To find this Multiplicator, divide an Unit with Cyphers by 25, the Diviſor propoſed, the Quotient will be .04, the Multiplicator ſought: For 900 being multiplied by .04, the Product will be 36, which was required.

Again, Having a Multiplicator, to find a Diviſor; this is but the Converſe of the former: For if an Unit with Cyphers be divided by .04, (a Multiplicator) the Quotient will be 25, the Diviſor as before.

6. A *Vulgar Fraction* is reduced into a Decimal by Diviſion: For which the Rule is,

Divide the *Numerator* (of the *Vulgar* Fraction given) by the Denominator, and the *Quotient* will be a Decimal Fraction, equal in Value to the *Vulgar* Fraction given: So $\frac{3}{4}$ reduced as aforeſaid, will be .75.

EXAMPLE.

4) 3.00 (.75
20
00

Note, the odd Parts of an Integer cannot be exactly reduced to a Decimal, for there will always be a Remainder; in ſuch Caſes carry on the Fraction to four or five Places, it will be very near the Truth.

EXAMPLE.

Remainder $\frac{4}{9}$ 9) 4.0000 (.4444
40
40
40

7. To reduce a Decimal Fraction to the known Parts of the Integer.

If the Decimal given be part of a Pound Sterling, multiply it by 20 (the Shillings in a Pound) and the Fractions in the Product by 12 (the Pence in a Shilling) and the Fractions in this Product by 4, (the Farthings in a Penny.) Then the whole Numbers in the reſpective Products ſhew the Shillings, Pence and Farthings contained in the Decimal given, and the Fractions in the laſt Product are Decimals of a Farthing. *Example*; Suppoſe .25 a Decimal Fraction of a Pound Sterling, were given to be reduced as aforeſaid; Multiply it by 20, the Number of Shillings in a Pound, the Product will be 5, *viz.* 5 *s.*

.25
20
Shillings 5.00

So the Value of .60625 will be found to be 12 *s.* $1\frac{1}{2}$ *d.* As appears by the following Example:

.60625

.60625
20

Shillings —— 12.12500
12

25000
12500

Pence —— 1.50000
4

Farthings —— 2.00000

If the Decimal given be part of a *Beer Barrel*, multiply it by 4, (the *Firkins* in a Barrel) and the Fractions in the Product by 9 (the Gallons in a Firkin,) and the Fractions in that Product by 8, (the Pints in a Gallon.) The *Whole* Numbers in the reſpective Products are equal to the *Firkins*, *Gallons*, and *Pints* contained in the Decimal propounded. *Example.*

Let .72 be a Decimal of a Beer-Barrel, to be reduced as aforeſaid, it will be found to be 2 Firkins, 7 Gallons, 7 Pints, and .36 of a Pint, as appears by the Work.

.72
4

Firkins —— 288
9

Gallons —— 7.92
8

Pints —— 7.36

But theſe and many other *Arithmetical* Queſtions may be more readily reſolved by the *Inſtrument*; a Deſcription of which ſee in the firſt Section immediately after this Introduction.

GEOMETRY.

DEFINITIONS.

1. GEOMETRY is that Science which teaches us to diſcover either the whole Magnitude, or ſome Part of any propoſed Quantity, by comparing it with another Quantity of the ſame Kind, which will always be either a *Line*, a *Superficies*, or a *Solid*.

2. A *Solid* is that which hath *Length*, *Breadth*, and *Depth*, or *Thickneſs*; its Limits or Boundaries being Superficies. As A B C D E F G (Fig. 1. Plate II.)

3. A *Superficies* hath *Length* and *Breadth* without *Thickneſs*; the Limits of which are Lines. As A B C D (Fig. 2. Plate II.)

4. A *Line* hath only *Length*, as A B (Fig. 3. Plate II.) and is bounded by Points; and therefore

5. A *Point Mathematical* is incapable of Diviſion, and, conſequently hath no Parts.

6. A *Right Line* is that which lies equally betwixt, or is the neareſt Diſtance between two Points; as A B. Fig. 3.

7. A *Plane Angle* is the Inclination of two Lines, the one to the other, and touching each other in the ſame Point, but not lying in the ſame ſtraight Line. The

Point where they meet is called the Angular Point, as A, Fig. 4. And according as the Lines are nearer or further from each other, the Angle is ſaid to be leſſer or greater.

8. When a Right Line, as C D (Fig. 5.) ſtands upon another Right Line, as A B, making the Angles A D C, and C D B equal to each other, then are both of theſe Angles termed Right Angles, and the Right Line C D is ſaid to be Perpendicular to the Line A B on which it ſtandeth.

N. B. *When ſeveral Lines meet at the ſame Point as at* D, (Fig. 5.) *each particular Angle is deſcribed by three Letters, the middle Letter denoting the Angular Point, and the other two Letters the Lines which form that Angle. Thus the Angle formed by the Lines* A D, *and* C D, *is called* A D C, *or* C D A.

9. An *Obtuſe Angle* is that which is greater than a Right Angle, as the Angle A D E (Fig. 5.)

10. An Acute Angle is that which is leſs than a Right Angle; and B D E (Fig. 5.)

11. A Limit or Term is the End or Boundary of any Thing.

12. A Figure is that which is contained under one or more Terms.

13. A Circle is a plain Figure comprehended under one Line, called the Periphery or Circumference; as A D B C (Fig. 6.) All Lines drawn from a certain Point within the Figure to the Circumference are equal to one another.

14. And that Point is called the *Center* of the *Circle*; as C (Fig. 6.)

15. The *Radius* of a Circle is a Right Line intercepted between the Center and the Circumference; as C D, or C A (Fig. 6.)

16. The *Diameter* of a Circle is twice its Radius, being a Right Line drawn through the Centre and terminated on either Side by the Circumference; as the Right Line A B (Fig. 6.)

17. A *Semi-circle* is a Figure which is contained under the Diameter, and that Part of the Circumference which is intercepted by the Diameter, as A C B D, (Fig. 6.)

Note. *The Circumference of every Circle is ſuppoſed to be divided into* 360 *equal Parts called Degrees, and each Degree into* 60 *equal Parts called Minutes, and each Minute into* 60 *equal Parts called Thirds,* &c. *Whence the Arch of every Semi circle contains* 180 *Degrees, and the Quadrant* 90.

18. A *Chord Line*, or Subtenſe of an Arch, is a Right Line which divides the Circle into two unequal Parts, as the Line F G (Fig. 6.) and is always leſs than the Diameter.

19. A *Segment of a Circle* is a Figure intercepted between the Chord and that Arch of the Periphery which is cut off by the Chord; and may be either Greater or Leſs than a Semi-circle; as the Figure F L G, or F D G (Fig. 6.)

20. A *Sector* is a Figure comprehended between two Radius's and that Part of the Circumference which is intercepted between thoſe Radii; thus the Figure I C D is called a Sector (Fig. 6.)

21. A Plain Triangle is a Figure whoſe Area is comprehended within the Limits of three Right Lines called Sides, including three Angles; and receives its Name according to its Sides or Angles.

22. An *Iſoſcecles Triangle* is that which has only two Sides equal; as A B C (Fig. 7.) That is A C = B C, but A B, may be of a different Length.

23. An *Equilateral Triangle*, is that which has all its three Sides equal, as A B C (Fig. 8.) That is $AB = AC = CB$.

24. A *Scalenum*, or *Scalenus Triangle*, is that whoſe three Sides are all unequal; as A B C (Fig. 9.)

25. A *Right-angled Triangle*, is that which hath one Right Angle; that is, when two of its Sides are perpendicular to each other, for then the Angle comprehended between them will be a Right-angle, by Def. 8. Thus A B C (Fig. 4.) is a Right-angled Triangle, becauſe the Sides A B and B C are perpendicular to each other.

N. B. *The longeſt Side of every Right-angled Triangle is called the* Hypothenuſe, *as* A C (Fig. 4.) *the longeſt of the other two Sides, which comprehend the Right-angle, is termed the* Baſe, *as* A B; *and the other, as* B C, *is called the* Cathetus *or* Perpendicular.

26. An *Amblygonum* or *Obtuſe-angled Triangle* is that which has one Angle Obtuſe, or greater than a Right Angle; as A B C (Fig. 9)

27 An *Oxygonum*, or *Acute-angled Triangle*, is that which hath three acute Angles; that is, each leſs than a Quadrant, or Right Angle, as A B C (Fig. 7.)

28. The *Altitude* or Height of any Plane Triangle is the Length of a Right Line, let fall Perpendicular from any Angle on the Side oppoſite to that Angle; and may be either within or without the Triangle, as Occaſion requires; thus the prick'd Line C D is the Altitude of the Triangle A B C (Fig. 8.) and C E the Altitude of the Triangle A B C, (Fig. 9.)

29. A *Square* is a Figure limited by four equal Sides, perpendicular to one another, as A B C D (Fig. 10.)

30. A *Rhombus*, is that which hath four equal Sides, but no Right Angle, as A B C D (Fig. 11.)

31. A *Parallelogram*, is a Quaderateral Figure, whoſe Sides are Parallel or Equidiſtant, as (Fig. 2, 11.) And

32. If the four Sides are Perpendicular to each other, it is called a Right-angled Parallelogram, as A B C D (Fig 2.) But,

33. If the four Sides are not Perpendicular, it is called an Oblique-angled Parallelogram, as A B C D (Fig. 11.)

34 The Altitude of an Oblique-angled Parallelogram, is a Right Line let fall Perpendicular from any Angle upon the Side oppoſite to that Angle; and may either be within or without the Figure, as the Right Lines C c, D d, (Fig. 11.)

35. All other four-ſided Figures, or thoſe which differ from the preceding, are termed Trapezia, or Tables, as A B C D (Fig. 12.)

36. A Right Line drawn from any Angle in a Trapezium, to its oppoſite Angle, is called a Diagonal, and divides the Figure into two Triangles, as the Right Line B C (Fig. 12.)

37. All Right-lined Figures which have more than four Sides, are called Polygons; and if all its Sides and Angles are equal, it is called a regular Polygon. And derives its Name from its Number of Sides, or Angles. That is, if it has five equal Sides, it is called a *Pentagon*; if ſix, a *Hexagon*; if ſeven, a *Heptagon*; if eight, an *Octagon*, &c. But if its Sides or Angles are unequal, it is called an irregular Polygon.

PROBLEMS.

POSTULATES OR PETITIONS.

I. To draw a Right Line from any one given Point to another.

II. To pruduce or increaſe a finite Right Line from either of its Ends.

III. Upon any Centre, and with any Radius to deſcribe a Circle.

PROBLEM I.

Upon a finite Line, as A B, (Fig. 8.) *to deſcribe an Equilateral Triangle.*

Make the given Line A B Radius (that is, take the Length of the given Line between the Points of the Compaſs) and ſetting one Foot in A, deſcribe the Arch B C, and removing it to B deſcribe the Arch A C, cutting the former in C; from whence draw two Right Lines A C, B C, and the Thing is done.

PROB. II.

To biſſect, or divide a given Right Line into two equal Parts.

Let the given Line be A B, (Fig. 13.) with any Radius greater than half A B, and on the Centres A and B deſcribe two Arches cutting each other in C and D; then join the Points C and D with a Right Line, which will divide A B into two equal Parts.

PROB.

PROB. III.

To draw a Right Line parallel to a given Right Line, that ſhall paſs through any Point aſſigned.

Let A B (Fig. 14.) be the given Line, and *x* the aſſigned Point. Take any convenient Place in the Line A B (the further from *x* the better) and with the Radius C *x* deſcribe the Semicircle *a b x d*; make the Arch *a b* equal to the Arch *d x*, and draw the Line D E through the Points *d x*, which will be Parallel to the Line A B, as was required.

PROB. IV.

To erect a Perpendicular upon the Middle of a Line.

Let the given Line be A B (Fig. 15.) and the Perpendicular be required to be erected from the Point C. Make C *f* and C *g* equal to each other; then with any convenient Radius greater than C *g*, and on the Centres *f* and *g* deſcribe the two Arches *a b*, *c d* interſecting each other in D; draw the Right Line D C, which will be the Perpendicular required.

PROB. V.

To erect a Perpendicular on the End of a Line.

Let the given Line be A B, (Fig. 16.) and the Perpendicular to be erected on the Point B. With any convenient Radius on the Centre B deſcribe the Arch *e f g*, and make *e f* and *f g* equal to the Radius *e* B; then with the ſame Radius, on the Points *f* and *g* deſcribe the Arches *a b*, *c d*; through the Interſection of thoſe two Arches draw the Right Line C B, which will be the Perpendicular required.

PROB. VI.

To deſcribe a Circle that ſhall paſs through three given Points, provided they are not ſituated in a Right Line.

Let the given Points be A, B, D, (Fig. 17.) join A B and A D with Right Lines, and biſſect thoſe Lines by PROB. II. continuing the biſſecting Lines till they croſs each other, which will be the Centre of the Circle required.

PROB. VII.

To deſcribe a Square about any given Circle.

In the given Circle draw two Diameters at Right Angles to each other, A B, and D E; (Fig. 18.) with the Radius of the Circle C B, deſcribe, from the extreme Points of the Diameters A, B, D, E, Arches interſecting each other in the Points F, G, H, I, join thoſe Points with Right Lines, which will form the Square required.

PROB. VIII.

To deſcribe the largeſt Square that can be contained in a given Circle.

Having drawn the Diameters A B, D E (Fig. 8.) as in the preceding Problem, join the Points A, E, B, D, with Right Lines, which will be the Sides of the Square required.

PROB.

PROB. IX.

Upon any given Right Line to deſcribe a Regular Pentagon.

Let the given Line be A B, (Fig. 19.) on each End of it, with the Radius A B, deſcribe a Circle, and through the Points where they interſect each other, as at C, *c*, draw the Right Line C *e c*; upon the Point C, with the ſame Radius, deſcribe the Arch D A *e* B E; lay a Ruler on the Points D, *e*, marking where it interſects the other Circle, as at G, alſo lay a Ruler on the Points E, *e*, obſerving where it cuts the other Circle as at F. On the Points G and F, with the ſame Radius, deſcribe the two Arches interſecting each other in H; join the Points A F, B G, G H, and F G, which will form the Pentagon required.

PROB. X.

The Length of the Tranſvers and Conjugate Diameters being given, to deſcribe or draw an Ellipſes or Oval.

Let the two given Diameters be G H, and I K, (Fig. 20.) Draw the Right Line A B equal in Length to G H; Biſſect it in L, and draw the Right Line C D, through the Point L, at Right Angles to A B, making L C or L D equal to half I K. Make L A or C B Radius, and on the Points D and C deſcribe the Arches *a* E F *d*, and *e* E F *b*, interſecting each other in the Points E and F. In thoſe Points fix two Pins or Nails, and putting a String or Thread round them of ſuch a Length that the two Ends, being faſtened together, may juſt reach to C or D. Faſten a black Lead Pencil, *&c.* in the Place where the two Ends of the Thread are joined; move the Pencil carefully round, and it will deſcribe the Ellipſis A B C D required.

STEREOMETRY:

OR, THE

Art of GAUGING made Eaſy,

By the HELP of a New

SLIDING RULE.

SECT. I.

A Deſcription of the RULE, *and of the ſeveral Lines upon it, with their Uſe in ſome Queſtions in* Arithmetic.

THE Rule conſiſts of Three Parts, *viz.* a Rule of 12 Inches long, and two ſmall Scales to ſlide in it, which may be drawn out, one towards the right Hand, and the other towards the left, till the whole be three Foot long. See Plate I.

The principal Lines on the Inſtrument are thoſe commonly known by the Name of *Gunter's-Line*, or

Line of Numbers, which are here diſtinguiſhed one from another by certain Letters, ſet at the End of the Lines towards the right Hand.

Thus, the Lines D, are each of them one ſingle Line of Numbers, beginning at the End of the Rule towards the left Hand, from thence continued to the other End.

The Lines A B, and C, are called double Numbers, each being two Lines or Radiuſes of Numbers, the firſt beginning at the left Hand, and ending in the Middle of the Rule where the ſecond Radius begins, and is from thence continued to the End at tha Right Hand.

The Line E is call'd Triple Numbers, being three Radius's of the Numbers; the firſt beginning at the left Hand, and the third ending at the right Hand.

This Triple Line is equal in Length to the double Lines, and all to the ſingle Line; for all the five begin and end at the ſame Point.

The Line of Numbers is (now) ſo well known to moſt Perſons, that it may be thought a ſufficient Deſcription to have only ſaid that theſe are ſuch.

But as this Inſtrument may be uſeful to ſome who (I preſume) do not yet know what the Line of Numbers is. I ſhall therefore endeavour to explain it as follows.

The Line of Numbers, is a Line of Geometrical Proportions, divided firſt into Nine unequal Parts, call'd *Primes*, which are diſtinguiſhed by Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; and then each of theſe *Primes* are ſubdivided, into ten other Parts (according to the ſame Reaſon) call'd Tenths, and again each of thoſe Tenths are ſubdivided, or at leaſt ſuppos'd to be ſubdivided into ten other Parts, according as the Length of the Line will admit; as here the Line D, being about 11 Inches long, each Tenth in the firſt *Prime* is really ſubdivided into *ten* Parts, called *Centeſms*, but betwixt the Figure 2 and 4, each *tenth* is divided but into five Parts, and therefore each of thoſe Parts ſignifies *two Centeſms :* Again, from the Figure 4 to the End of the Line every Tenth is divided into two Parts, each repreſenting five *Centeſms :* Laſtly, each of theſe, *Centeſms* is alſo ſuppoſed to be divided into *ten* Parts which by ſome are called *Millions ;* but a Line of this Length will not admit of this laſt Diviſion.

The Figures (1, 2, 3, 4, 5, 6, *&c.*) by which the *Primes* are diſtinguiſh'd, are all arbitrary Points, and may any of them repreſent ſo many entire *Units, Tens, Hundreds,* or *Thouſands ;* or they may alſo repreſent ſo many *Tenth, Hundredth, Thouſandth,* or *Ten Thouſandth* Parts of an *Unit*.

1. For *whole*, or *intire Units ;* Let the firſt *Prime* or the Fig. at the Beginning of the Line D, repreſent one *Unit,* then ſhall all the Figures towards the right Hand, (*viz.* 2, 3, 4, 5, *&c.* to 10.) repreſent *ſo many* Units, and the Tenths in each *Prime*, will be *tenth Parts*, and the *Centeſm* in each of thoſe Tenths will be *hundredth Parts* of an Unit.

Or, let 1 at the Beginning of the Line repreſent 10 Units, then will each *Prime* forwards repreſent 10 times ſo many Units as the Figures expreſs ; thus, the Figures 2, 3, 4, 5, *&c.* will be 20, 30, 40, 50, *&c.*

And when one *Prime* repreſents 10 *Units*, every *tenth* in that *Prime* will be 1 *Unit*, and each *Centeſm* in thoſe *tenths* will be 1 tenth Part of an *Unit*.

Again, Let the firſt *Prime* repreſent 100, then the Figures 2, 3, 4, 5, *&c.* will repreſent 200, 300, 400, 500, *&c.* and therefore 10 at the End will be 1000 ; and according to this Suppoſition, 1 Tenth in each *Prime* will be 10 Units, and in thoſe Tenths each *Centeſm* will be 1 Unit.

2. For *Decimal Fractions*; Let 10 at the End of the Line (at D) repreſent .1, then each *Prime* towards the left Hand will be .1, and in thoſe *Primes* each Tenth will be .01, and theſe Tenths each *Centeſm* will be .001, Part of an Unit.

To make this more plain, draw out the ſliding Piece B, till 1 at the *Beginning* of the Line B, ſtand exactly againſt 10 at the *End* of the Line A; for then you have a Line of Numbers four times repeated; upon which, let 1 at the Beginning of the Line A, repreſent 1 Unit; Then ſhall 1 in the *Middle* of the ſaid Line be 10; and 10 at the *End* thereof (or which is all one, at the *Beginning* of the Line B,) repreſent 100; and by Conſequence 1 in the *Middle* of the Line B will be 1000, and for the *ſame* Reaſon 10 at the End of the ſaid Line (which is alſo the *End* of the *Rod*) will be 10000.

But keeping the Rule as it now ſtands, let 10 at the End of the fourth Radius (*viz.* at B.) repreſent 1, then ſhall each Prime in the ſaid fourth Radius repreſent .1, in the third Radius .01, in the ſecond Radius .001, and in the firſt .0001 Part of an Unit.

So will 2 in the firſt Radius be .0002, in the ſecond .002, in the third .02, and in the fourth .2 Parts.

The *Numbers* and *Diviſions* on the Lines being thus explained, it will not be difficult to find the Point upon the Line, where any Number given is repreſented.

As for Example; ſuppoſe the Numbers were 1895, for the firſt Figure thereof (*viz.* 1.) I count 1 at the Beginning of the Line D, for the ſecond Figure I count 8 Tenths next following (that is 8 of the greater Diviſions betwixt 1 and 2) then from this Point forward I count 9 *Centeſms* for the third Figure, and for the laſt Figure 5, I count the next Centeſm; ſo I find the Point *a g*, will repreſent 1895, and by the ſame Rule, the Number 1715 will be found at the Point *w g*. Hence obſerve,

1. That on a Line of this Length, only the four firſt Figures of any Number propos'd can be diſcovered; for if the Number given were 189582, it would be repreſented at the ſame Point where the former 1895 was found, *viz.* at *a g*.

2. That all Numbers, which, after the firſt Figure have nothing but Cyphers (as, 20, 200, 2000, *&c.*) are all repreſented at the ſame Point.

So the Numbers 20, 200, 2000, are all repreſented by the *Figure* 2, at the Beginning of the ſecond *Prime*.

3. All Numbers conſiſting of three *Figures*, and having a *Cypher* in the Middle, are found within the firſt Tenth of that *Prime*, at which the firſt Figure of the Number given is found.

Example. Let the Number given be 308; for the firſt Figure I count 3 on the Line (which I find at the Beginning of the third *Prime*.) Now there being a *Cypher* in the *ſecond* Place, I muſt not count any of the Tenths; but for the *laſt Figure* I count 8 *Centeſms*, and that is the Point which repreſents 308.

4. All Numbers conſiſting of *four* Places, and having two Cyphers in the Middle, muſt be ſought betwixt the Beginning of the *Prime* unto which they belong, and the firſt *Centeſm* of the ſame Prime: So 4005 being given, the *firſt Figure* (viz.) 4, is found at the Beginning of the fourth Prime. Now there being Cyphers in the ſecond and third Places, I muſt not count any of the Tenths or *Centeſms*; but for the *laſt Figure* 5, I eſtimate 5 Millions, which is about the Middle of the firſt *Centeſm*, and that is the Point where 4005 is repreſented.

Note, *Decimal Fractions* and *Mixt Numbers* are diſcovered after the ſame manner as whole Numbers; for if the firſt Number 1895, were 18.95, it will be found at the ſame Point, *viz.* at *a g*; and by what hath been ſaid, it is eaſy to find what Number is repreſented at any Point upon the Line; as will appear in the following Queſtions of Proportion, of which I come now to treat.

In *Arithmetic* I ſhall take notice of two Sorts of Proportion, *Arithmetical* and *Geometrical.*

1. *Arithmetical* Proportion is when divers Numbers, being compared together, retain amongſt themſelves equal Differences; as theſe, 2, 4, 6, 8, *&c.* And this is either *continued,* as in the Numbers before propounded (which are alſo called *Arithmetical Progreſſion,* or a Rank of Numbers *Arithmetically* proportional;) or *diſcontinued,* as in theſe, 2, 4, 8, 10, or the like.

2. *Geometrical* Proportion, is, when divers Numbers being compared together, differ among themſelves, according to the ſame Ratio or Reaſon, as theſe, 2, 4, 8, 16, *&c.* for here, as 2 is half 4; ſo four is half 8, and 8 half 16. This is likewiſe either *continued,* as in thoſe before propounded, (which are alſo called *Geometrical Progreſſion,* or a Rank of Numbers *Geometrically* proportional;) or *diſcontinued,* as in theſe, 2, 4, 16, 32; for as 4 is double 2; ſo is 32 double 16; but ſo is not 16, being compared with 4.

PROB.

PROBLEM I.

Having two Numbers given, to find a third Geometrically proportional unto them, and to three a Fourth, and to four a Fifth, &c.

FInd *one* of the Numbers given upon the Line B, and ſet it againſt the *other* given Number on the Line A; then find the ſame Number upon B (which was laſt counted upon A) and againſt it you have this *third* proportional upon A; and againſt this *third* upon B, is the *fourth* upon A. In like Manner, againſt the *fourth* upon B, you have the *fifth* on A, *&c.*

Example. Let it be required to find a *third* Proportional to theſe two Numbers 2 and 4, which may bear the ſame Proportion to 4, that 4 bears to 2.

Draw out the Sliding Rod, till 2 upon B ſtand againſt 4 upon A; then againſt 4 upon B, is 8 (the *third* Proportional) upon A; and againſt this *third* (*viz.* 8) upon B, is 16 upon A; which is the *fourth* Proportional. Likewiſe againſt 16 upon B, is 32, the *fifth* upon A; and againſt 32 upon B, is 64, the *ſixth* Proportional. But now proceeding forward, I find that 64 upon B, will reach beyond the End of the Line A; I therefore ſeek 64 towards the left Hand upon B; and againſt it I find 128, the *ſeventh* Proportional: And ſo proceeding further, you may find the *eighth* to be 256, the *ninth* 512, *&c.* *Contrariwiſe*, if it were required to find a *third* Proportional to the ſame Numbers 2 and 4, which may bear the ſame Proportion to 2, that 2 bears to 4, Set 4 in the ſecond Radius upon A, to 2 upon B; then againſt 2 upon A (towards the left Hand) is 1, the *third* Proportional; and againſt 1 upon A is .5, the *fourth* upon B: Alſo againſt this *fourth* (*viz.*) upon A, is .25, the *fifth* Proportional on B, *&c.*

Multiplication by the Lines.

PROB. II.

One Number being given to be multiplied by another, to find the Product.

IN Multiplication, either or whole Numbers, mixt or decimal *Fractions*, the Proportion is,

As 1 *is to the Multiplicator;*
So is the Multiplicand to the Product.

And the Product of any two Numbers ſhall have ſo many Places as there be in both the Numbers given, except when the leſſer of them do not exceed ſo many of the firſt Figures of the Product; then it will have One leſs.

1. Example. *Let it be required to multiply* 6 *by* 4.

Say, [1 : 4 : : 6 : 24] which Analogy or Proportion may be read thus: *As* 1 is to 4, *So is* 6 *to* 24.

Therefore, Set 1 upon the Line B, to 4 on the Line A; and then againſt 6 upon B, is 24 upon A; which is the Product ſought.

Note, The Unit or firſt Term may be taken upon either of the Lines A or B; but the firſt and third Terms muſt be counted upon *one* and the *ſame* Line, and the *Second* on the other Line, where the *fourth* will alſo be found.

The Letters A and B may ſerve to diſtinguiſh the Lines.

2. Example. *Let the two Numbers given be* 68 *and* 26, *to find the Product.* The Proportion is [1 : 26 : : 68 : 1768]

Therefore Set 1 upon B, to 26 upon A; and then againſt 68 upon B, is 1768 on A, which is the Product ſought. Or,

Set 1 upon B, to 68 upon A; then againſt 26 upon B, is 1768 (on A) the Product. Therefore it matters not which of the Numbers given be made the Multiplicator: And note alſo, that the Product hath as many Places as are in both the Numbers given; becauſe the leaſt of them (*viz.* 26) exceeds ſo many of the firſt Figures of the Product, according to the Rule before given.

3. Example. *Let* 68 *be multiplied by* 14. The Proportion is, 1 : 14 : : 68 : 952.

Therefore Set 1 upon B, to 14 upon A; then againſt 68 upon B, is 952, the Product upon A; and here the Product conſiſts of one Place leſs than there is in the two Numbers given; becauſe the leſſer of them (*viz.* 14.) doth not exceed ſo many of the firſt Figures of the Product.

Now, that the Product laſt found, is 952, and not 95.2, nor 9.52, will this appear: Set 1 upon B, againſt 14 upon A,

And then againſt		upon A is		upon B
	2		28	
	3		42	
	4		56	
	5		70	
	6		84	
	6.8		95.2	

By this 'tis evident, that the Lines are in effect a Table of Multiplication; for having ſet ,1 to the Multiplicator, againſt any Multiplicand, you have the Product: So if 2 be multiplied by 14, the Product is 28; if 3, the Product will be 42; if 4, the Product will be 56; if 5, 70, *&c.* Hence I conclude, that if the Multiplicand had been but 6.8, the Product would have been 95.2; but the Multiplicand being 68, the Product muſt be 952; for by taking away the Prick, the Fractions in each are made *whole Numbers.*

When of *two Numbers* given to be multiplied, the one conſiſts of *whole Numbers* or *mixt*, and the other of *Fractions* only, make the Whole or Mixt Number the Multiplicator; and having ſet 1 againſt it, ſeek the Fraction towards the left Hand, and againſt it you have the *Product.*

4. Example. *Let the two Numbers be* 27.5, *and* .8.

Set 1 upon A, to 27.5 on B, and then againſt .8, (which being leſs than 1, I ſeek towards the left Hand) on A, is 22, the Product on B.

And notwithſtanding a Number of more than four Places cannot be exactly expreſſed on a Line of this Length, yet the Product of any Multiplication may be diſcovered to ſix or ſeven Places at leaſt.

5. Example. *Suppoſe I were to multiply* 1482 *by* 54. The Proportion is,

1 : 54 : : 2482 : 134028.

Therefore ſet 1 upon A to 54 upon B; and then againſt 2 upon A, is 108 upon B. Now, ſuppoſe,

$$2 \text{ be } \left\{\begin{array}{l}20\\200\\2000\end{array}\right\} \begin{array}{c}108\\ \text{will be}\end{array} \left\{\begin{array}{l}1080\\10800\\108000\end{array}\right\}$$

So the Product will have ſix Places; and againſt 2842, the Multiplicand, you may diſcern (upon the Line) the four firſt of them, (*viz.*) 1340; the two laſt may be found by multiplying (in ones Mind) the two laſt Figures of the Multiplicand, by the two laſt of the Multiplicator; by which means the two laſt Figures of the Product may be found, which in this Example will be 28; theſe being placed behind the four firſt, *viz.* 1340, makes the Product complete 134028.

I might here add more Examples; but theſe already given, may ſerve, they containing (I think) ſufficient Directions for all the Variety that can happen in Multiplication.

DIVISION *by the* LINES.

PROB. III.

One Number being given to be divided by another, to find the Quotient.

IN Diviſion both of whole Numbers and mixt, the the Proportion is,
As the Diviſor is to 1;
So is the Dividend to the Quotient.

Which Quotient ſhall ever conſiſt of ſo many Figures as the Dividend hath more than the Diviſor, except when the Diviſor does not exceed ſo many of the firſt Figures of the Dividend; then it ſhall have one Place more.

1. Example. *Let it be required to divide* 24 *by* 4, The Proportion is, 4 : 1 : : 24 : 6.

Therefore, Set 4 upon B, to 1 upon A; and then againſt 24 upon B, is 6 upon A, which is the Quotient ſought.

2. Example. *Let it be required to divide* 1768 *by* 26. The Proportion is, 26 : 1 : : 1768 : 68.

Therefore, Set 26 upon B, to 1 upon A; and then againſt 1768 upon B, is 68, (the Quotient) upon A.

3. Example. *Suppoſe* 952 *were to be divided by* 14. The Proportion is, 14 : 1 : : 952 : 68.

Therefore, Set 14 upon A, and 1 upon B, and againſt 952 upon A, you have 68 upon B, which is the Quotient required.

Obſerve here, the Dividend hath but one Figure more than the Diviſor; yet the Quotient conſiſts of two Figures; becauſe the Diviſor doth not exceed ſo many of the firſt Figures of the Dividend; but in the firſt and ſecond Examples the Quotients have but ſo many Figures as the Dividend hath more than the Diviſor; becauſe the Diviſor doth not exceed ſo many of the *firſt Figures* of the Dividend, according to the general Rule above given.

This ſhews (in all Caſes) how many Figures muſt be in the Quotient, the Value of the firſt of which may be found by the Rule given in the Introduction. Page 8.

Or, it will thus appear, that the Quotient in the laſt Example is 68, and not 6.8, nor .68: Set 14 upon A, to 1 upon B,

And againſt	28	upon A is	2	upon B
	42		3	
	56		4	
	70		5	
	84		6	
	95.2		6.8	

This is but the Converſe of the third Example of Multiplication; and as by that it is manifeſt, that if 2 were multiplied by 14, the Product would be 28; if 3, it would be 42; if 4, 56, &c. So here it is evident, that if 28 be divided by 14, the Quotient will be 2; if 42, it will be 3; if 56, it is 4, &c. Thus by reading on the Proportion from the Diviſor 14: 28, 56,

70,

70, *&c.* I find at laſt, that if the Dividend were but 95.2, the Quotient muſt be 6.8; but the Dividend being 952, the Quotient muſt be 68; for by taking away the Prick, the Fractions in each are made whole Numbers.

By theſe Examples it is alſo evident, that at once ſetting of the Rule, we both multiply and divide.

For if 14 be a Multiplicator, ſet 1 on B againſt 14 on A; then againſt any Multiplicand upon B, you have the Product upon A, as appeared by the third Example of Multiplication.

And without moving the Rule, if you ſuppoſe 14 to be a Diviſor, then againſt any Dividend upon A, you have the Quotient upon B; as in the laſt Example of Diviſion.

How by any Diviſor to find a Multiplicator, was ſhewn in Page 13. of the *Introduction*; it may alſo be performed by the *Lines* more readily.

Thus, having a *Diviſor*, to find a *Multiplicator*.

Set the Diviſor given upon A, to 1 upon B; and then againſt 1 (towards the left Hand) upon A, is the Multiplicator upon B. *Example.* Suppoſe 25 were a Diviſor, given to find a Multiplicator.

Set 25 upon A, to 1 upon B; and then againſt 1 (towards the left Hand) upon A, is .04, the Multiplicator ſought.

By a Multiplicator to find a Diviſor. This is but the Converſe of the former; for having .04, a Multiplicator upon B, ſet it againſt 1 upon A; and then againſt 1 upon B, is 25, the Diviſor, as before.

PROB.

PROB. IV.

To reduce a Vulgar Fraction into a Decimal.

SET the Denominator (of the Vulgar Fraction given) upon A, to the Numerator thereof upon B; and then againſt 1 towards the left Hand upon B, is the *Decimal Fraction* ſought.

So $\frac{1}{4}$ will be found equal to this Decimal (*viz.*) .25; for,

As 4 upon A, is to 1 upon B; *So* is 1 upon A, to .25 upon B.

Alſo $\frac{21}{28}$ will be .75; for *As* 28 upon A, is to .21 upon B; *So* is 1 upon A to .75 upon B.

In like manner, this Vulgar Fraction (*viz.* $\frac{582}{960}$,) is equal to this Decimal, *viz.* .60625; for ſet 960 upon A, to 582 upon B; then againſt 1 upon A, is .60625 upon B; and ſo of any other.

PROB. V.

A Decimal Fraction being given, to reduce the ſame into the known Parts of an Integer.

1. IF the Decimal be Part of a Pound Sterling, Set 1 upon B, to the Number of Shillings, Pence, or Farthings contained in a Pound, upon A; then ſeek the Decimal given upon B, (towards the left Hand,) and againſt it you have the Shillings, Pence or Farthings reſpectively contained in the Decimal given.

Example. *Suppoſe .7625 were to be reduced into Shillings.*

1. Set 1 upon B, to 20 (the Number of Shillings in a Pound) upon A, then againſt .7625, upon B, (towards the left Hand) is 15.25, that is, 15 *s.* and 25 hundred Parts of a Shilling, which is equal to 3 *d.*

2. To reduce the Decimal given into Pence: Set 1 upon B, to 240 (the Pence in a Pound) upon A, then againſt .7625 toward the left Hand upon B, is 183, the Pence contained in .7625.

3. But if the Decimal aforeſaid were to be reduced into Farthings,

Set 1 upon A, to 960 (the Farthings in a Pound) upon B, and then againſt .7625 upon A, is 732; and ſo many Farthings are contained in .7625.

Again, ſuppoſe .7625 were a Decimal of an Ale-Barrel, to be reduced into Gallons and Pints.

Set 1 upon A, to 32 (the Gallons of an Ale-Barrel) upon B, then againſt .7625 upon A, is 24.4, that is, 24 Gallons, and 4 tenth Parts of a Gallon; to reduce this .4 into Pints; ſet 1 upon A, to 8 upon B, then againſt 4 upon A, is 3.2 Pints: So .7625 of an Ale-Barrel is equal to 24 Gallons, 2 Pints, and 2 tenth Parts of a Pint.

PROB. VI.

Three Numbers being given, to find a Fourth in a direct Proportion.

THIS is called the *Rule of three Direct*, and the Inſtrument is wrought thus;

Set the firſt Number given upon B, to the ſecond upon A, and then againſt the third Number given upon B, is the fourth Number ſought.

Example. *If 8 Quarters of Malt will make 20 Barrels of Small Beer, how many Barrels of ſuch Beer will 22 Quarters make?*

Set 8 upon B, to 20 upon A; and then againſt 22 upon B, is 55 upon A; and ſo many Barrels will 22 Quarters make: And according to this Proportion, 24 will make 60 Barrels, 28 will make 70, 32 Quarters will make 80 Barrels, *&c.*

PROB. VII.

To three Numbers given, to find a fourth in an inverſed Proportion.

THIS is called the *Rule of Three Inverſe*, in which obſerve, that if the third Number be greater than the firſt, then will the fourth be leſs than the ſecond. And, on the contrary, if the third Number be leſs than the firſt, the fourth will be greater than the ſecond; and in either Caſe the Rule is this:

Set the third Number upon A, the firſt (being of like Denomination) upon B; and then againſt the ſecond Number upon A, you have the fourth upon B.

Example. *If 8 Men do any Piece of Work in 9 Days; In how many Days can 12 Men do the ſame Work?*

Set 12 upon A, to 8 upon B; then againſt 9 upon A, is 6 upon B, which is the Anſwer. For 12 Men may do the ſame Work in 6 Days, that 8 Men will do in 9 Days.

But if the Queſtion had been, *In how many Days can ſix Men do the ſame Work?* The Anſwer will be, 12: for,

As 6 upon B, is to 8 upon A; *So* is 9 upon B, to 12 upon A.

PROB. VIII.

Betwixt two Numbers given, to find a Mean Geometrical Proportional.

SET one of the Numbers given upon C, to the ſame Number upon D; and then againſt the other given Number upon C, is the Geometrical Mean ſought.

Example. *Let the Numbers given be* 50 *and* 72, *to find a Geometrical Mean*, &c.

Set 50 upon C, to 50 upon D; and then againſt 72 upon C, is 60 upon D. So 60 is the Geometrical Mean betwixt 50 and 72. Or thus:

Set 72 upon C, to 70 upon D; and then againſt 50 upon C, is 60 upon D, the Mean, as before.

By the Pen thus:

Multiply one Number by the other; then extract the Square Root of the Product; this Square Root is the Geometrical Mean between the two Numbers given. Thus, 72 multiplied by 50, is 3600, whoſe Square Root is 60, the Geometrical Mean between 50 and 72.

PROB.

PROB. IX.

To find the Square Root of any Number under 1000000.

THE *Extraction of Roots* is one of the hardeſt Leſſons in Arithmetic; yet by the Help of this Inſtrument it may be performed with leſs Trouble than any of the foregoing Problems: For if the Lines C and D be applied one to another, ſo as 10 at the End of D be even to 10 at the End of C; I ſay the Lines thus applied, are like a Table ſhewing the Square Root of any Number by Inſpection only; for againſt any Number upon C, you have the Square Root thereof upon D, *& contr.*

Note, When the *Number* given conſiſts of 1, 3, 5, or 7 Places of Integers, ſeek it in the firſt Radius on the Line C; and againſt it, you have the Root required upon D.

Example. *Let the Number given be* 144. *I find this on the firſt Radius in the Line* C; *and againſt it is* 12 *the Root ſought.* In like manner,

Againſt	576	in the firſt *Radius* upon C, is	24	the Root upon D.
	20736		144	
	1000000		1000	

2. When the *Number* given conſiſts of 2, 4, 6, or 8 Places of Integers, find it in the ſecond Radius upon the Line C; and againſt it you have the Root upon D.

Example. *Let* 16 *be the Number given.*

I ſeek 16 in the ſecond Radius upon C, and againſt it is 4, the Root upon D. In like manner,

Againſt	30,25 2304 784996	in the 2d Rad. upon C, is	5.5 48 886	the Root upon B.

PROB. X.

To find the Cube Root of any Number under 1000.000.000.

PLace the Lines D and E, ſo as 10 at the End of D, be even with 10 at the End of E; then againſt any Number upon E, you have the Cube Root thereof upon D. *Et contr.*

Note, 1. When the Number given conſiſts of 1, 4, or 7 Places of Integers, find it in the firſt Radius of the Line E; and againſt it you have the Cube Root ſought. *Example.* Let the Number given be 3375. I ſeek this in the firſt Radius in the Line E; and againſt it I find 15 upon D, which is the Cube Root of 3375; and ſo is 212 the Cube Root of 9528128.

2. When the Number given conſiſts of 2, 5, or 8 Places of Integers, find it in the ſecond Radius upon E; and againſt it is the Root ſought. *Example.* Suppoſe 35.937 were propounded; find this in the ſecond Radius upon E; and againſt it is 3.3, the Cube Root upon D. In like manner, is 275, the Cube Root of 2.796875.

3. When the Number given conſiſts of 3, 6, or 9 Places of Integers, ſeek it in the third Radius, *&c.* and againſt it is the Cube Root. Thus againſt 125 in the third Radius upon E, I find 5 the Cube Root, and ſo likewiſe is 888 the Cube Root of 700.227.072.

Laſtly, to know how many Places of Integers muſt be in the Cube Root of any Number given.

Put a Point over the Place of Units in the Number given, then omitting 2, point over every third Figure towards the left Hand; then tell how many Points, for ſo many Places of Integers muſt the Cube Root conſiſt of.

SECT. II.

Of a SUPERFICIES.

. A *Superficies* is a Figure bounded by a Line or Lines, and is either *Round* or *Angular*.

2. A *Round Figure* is that which is contained by one ound Line, and is either a *Circle* (as Fig. 1.) or an *El-ipſis* (as Fig. 2.) Plate III.

3. An *Angular Figure* is that which conſiſts of three r more Angles, from the Number of which they are enominated, as a Figure of three Angles is called a *riangle*; of four, a *Quadrangle*, &c.

4. A *Triangle* is a Superficies comprehended by three ight Lines (as Fig. 3.) of theſe there are ſix Varieties, hich I ſhall forbear to mention, they being all meaſured y one and the ſame Rule.

5. A *Quadrangle* is a Figure comprehended by four Right Lines, and is either a *Parallelogram* or *Trapezium.*

6. A *Parallelogram* is a Figure whoſe oppoſite Sides are parallel, having equal Diſtances from one another in all Places, and is either Right or Oblique.

7. A *Right-angled Parallelogram*, is that whoſe Angles are all Right, and is either a *Square,* (as Fig. 5.) or an *Oblong,* (as Fig. 4.)

8. The *Oblique-angled Parallelogram,* is that whoſe Angles are all Oblique, and is either a *Rhombus* (as Fig. 7.) or a *Rhomboides,* (as Fig. 6.)

9. A *Trapezium* is a Quadrangular Figure, in which no two Sides are equal and parallel, (as Fig. 8.)

10. *Figures* conſiſting of *more* Sides than *four,* are almoſt innumerable, but are reducible into *two* Sorts, *Regular* and *Irregular,* either of which are alſo called *Polygons.*

11. *Regular Polygons* are ſuch, whoſe Sides and Angles are equal; they take their Names from the Numbers of their Sides, as that of five Sides is called a Pentagon (as Fig. 9.) that of ſix Sides, an Hexagon, &c. Of Irregular Polygons 'tis needleſs to ſay any Thing, they being meaſured after the ſame manner as a Trapezium.

I ſhould next proceed to ſhew the Uſe of the Rule in the meaſuring of Lines, Superficies, and Solids: But in order to this I muſt premiſe,

1. That every Magnitude muſt be meaſured by ſome known Kind of Magnitude that is Homogeneal or like to it. A Line is meaſured by a Line, as one Lineal Inch, or Foot, or Yard, *&c.*

A Superficies is meaſured by a Superficies, as one Square Inch or Foot, *&c.* A Solid is meaſured by a Solid, as one Cubic Inch, one Cubic Foot, *&c.*

And when it is known,

How many	Lineal	Inches or Feet are contained in a	Line,
	Square		Superficies,
	Cubical		Solid.

Then is the Quantity or Content of either of theſe Kind of Magnitudes ſaid to be known.

2. That the Meaſures of *Capacity* commonly uſed in *England* are of two Sorts, one for wet Commodities, as Beer, Cyder, Wine, *&c.* the other for dry Commodities, as Corn, *&c.*

And, I here take it for granted, that there are three ſeveral Gallons, whoſe Contents are allowed to be as followeth:

Ale	Gallon	282	Solid Inches.
Wine		231	
Corn		268.8	

Now from hence are derived the following Tables, ſhewing how many Solid Inches are contained in any of the other Meaſures reſpectively contained in them.

A TABLE of Beer-Meaſure.

Inches.						
35¼	*Pint.*					
70½	2	*Quart.*				
282	8	4	*Gallon.*			
2538	72	36	9	*Firkin.*		
5076	144	72	18	2	*Kilderkin.*	
10152	288	144	36	4	2	*Barrel.*

A TABLE of Ale-Meaſure.

Inches.						
35¼	*Pint.*					
70½	2	*Quart.*				
282	8	4	*Gallon.*			
2256	64	32	8	*Firkin.*		
4512	128	64	16	2	*Kilderkin.*	
9024	256	128	32	4	2	*Barrel.*

By an Act of the Firſt of *William* and *Mary*, 34 Gallons is the Barrel, both for Beer and Ale, in all Places, except within the weekly Bills of Mortality.

A TABLE of Wine-Meaſure.

Inches.								
$28\frac{7}{8}$	*Pint.*							
$57\frac{3}{4}$	2	*Quart.*						
231	8	4	*Gallon.*					
4158	144	72	18	*Rundle.*				
14553	504	252	63	$3\frac{1}{2}$	*Hogſhead.*			
19404	672	336	84	$4\frac{2}{3}$	$1\frac{1}{3}$	*Tericon.*		
29106	1008	504	126	7	2	$1\frac{1}{2}$	*Pipe.*	
58212	2016	1008	252	14	4	3	2	*Tun.*

This Table ſhews, that in 1 Tun there are 2 Pipes, 3 Tercions, 4 Hogſheads, 14 Rundlets, 252 Gallons, 1008 Quarts, 1016 Pints, and 58212 ſolid Inches. The Tables for *Beer* and *Ale* are like this, and need no Explanation.

SECT. III.

The Uſe of the Rule in meaſuring of Superficies; and firſt, of a Circle.

THE Area of the Superficial Content of a Circle is found by the Diameter or Circumference; I ſhall therefore firſt ſhew how by either of theſe to find the other.

PROB. I.

The Diameter, or Circumference of a Circle, either being given, to find the other.

THE Circumference of that Circle whoſe Diameter is Unity (or 1) is* 3.1415926536; but for our Purpoſe 3.141592, will ſuffice: Therefore, as 1 is to 3.141592, ſo is the Diameter of any Circle to the Circumference. By the Inſtrument thus:

Set 1 on the Line A, againſt 3.141592 on the Line B; then againſt any Diameter on the Line A, you have the Circumference on the Line B; and the contrary, thus: Againſt

* Foſter's *Problemata Geomet. Varia, Propoſ.* 2.

Theſe Diameters.	You have theſe Circumferences,
20	62.851
30	94.247
40	125.663
50	157.079

Or, Contrariwiſe, againſt

Theſe Circumferences	You have theſe Diameters,
20	6.366
30	9.549
40	12.732
50	15.915

And ſo of any other.

PROB. II.

The Diameter of any Circle being given, to find the Area, (or any Part thereof) in Inches or in Ale or Wine-Gallons.

1. *For the whole Area in Inches.*

THE Area of a Circle is equal to the Product, or Rectangle of half the Diameter into half the Circumference; that is, if half the Diameter be multiplied by half the Circumference, the Product will be the Area.

Thus, when the Diameter is 1, the Circumference is 3.141592, the half of this is 1.570796, which multiplied by half the Diameter (*viz.* .5) the Product will be the Area of that Circle, whoſe Diameter is 1, (*viz.*) .785398.

The Area's of all Circles are in Proportion one to another, as the Squares of their Diameters (2.12. of *Euclid.*)

Therefore, *As the Square of the Diameter of any Circle is to the Area of that Circle ;*

So is the Square of the Diameter of any other Circle to the Area thereof.

In the Circle above-mentioned the Diameter is 1, and the Area .785398 ; Now the Square of 1 being but 1, it muſt hold : As 1 is to .785398, So is the Square of the Diameter of any Circle to the Area thereof. So .785398 is a fixed Multiplicator ; and if an Unit with Cyphers be divided by .785398, the Quotient will 1.27324, a fixed Diviſor ; and by either of theſe fixed Numbers the Area of any Circle may be found, either by Multiplication or Diviſion. For if the Squares of any Diameter be,

$$\begin{array}{l}\text{Multiplied}\\ \text{Divided}\end{array}\left\{\text{by}\right\}\begin{array}{l}.785398\\ 1.27324\end{array}\left\{\begin{array}{l}\text{Prod.}\\ \text{Quot.}\end{array}\right\}\text{will be}$$

the Area in Square Inches, Feet or Yards, according as the Diameter was meaſured by Inches, &c.

But with more Expedition by the Inſtrument, Thus :

Set 1 (a Diameter) upon the Line D, to .785398 (the Area thereof) upon C.

The Rule being thus ſet, the Lines are like a Table of Circles Area's to all Diameters ; for againſt any Diameter upon the Line D, you have the Area thereof upon C.

Example: *Let the Diameter be* 20.

Set 1 upon D. to .785398 upon C, and then againſt 20 upon D, is 314.159, the Area required upon C; and as the Rule now ſtands, I alſo find, that if

The Diameter is		the Area is	
	25		490.87 Inches.
	30		706.85 Inches.
	40		1256.63 Inches.

On the Contrary, when the

Area is		the Diameter is	
	300		19.54
	400		22.56
	500		25.23

2. *For the Area in Gallons.*

The Area in Inches divided by 282 or 231, gives the Area in Ale or Wine-Gallons reſpectively, and ſo for any other Meaſure expreſſed in the former Tables; but without knowing the Area in Inches, the Area in Gallons may be found thus: Divide .785398.

By		the Quotient will be		
	282		.0027851	A. G.
	231		.0033999	W. G.

The Quotients are the Area of a Circle, whoſe Diameter is 1, in Ale and Wine-Gallons, and are fixt Multiplicators for finding the Area of all Circles in either of theſe Meaſures; for if the Square of the Diameter of any Circle be multiplied by either of theſe Numbers, the Product is the Area in Ale or Wine-Gallons reſpectively.

If you would effect this by Diviſion, the ſeveral Diviſors are thus found: Multiply the Diviſor for finding the Area's in Inches, *viz.* 1.27324.

By { 282 / 231 } the Product / will be { 359.05 / 294.11 } A. G. / W. G.

The Products are the Diviſors ſought. And the Square of the Diameter of any Circle divided by one of theſe, gives the reſpective Area.

Having thus found the ſeveral fixed Multiplicators and Diviſors for finding the Area of a Circle in Inches and Gallons, I ſhall repreſent them all together in the following Table.

	Multiplicators.	Diviſors.
Inches.	.785398	1.27324
Ale-Gallons.	.0027851	359 054
Wine-Gallons.	.0033999	294.118

But the Area of any Circle may be more readily found by the Help of certain fixt Numbers, called *Gage-points*; and theſe fixt Numbers are the Diameters of thoſe Circles, whoſe Content at one Inch deep is equal to the reſpective Gallon to which they belong. Thus, the *Gage-point* for the Ale-Gallons is 18.95, which is the Diameter of that Circle, whoſe Content is 282, the ſquare Inches in the Ale-Gallon.

The ſeveral *Gage-points* are the Square Roots of the Diviſions laſt mentioned, and by the Rule are all found at once by the Lines C and D; for (ſetting theſe Lines even at the end)

Againſt

Againſt theſe Diviſ. upon C.	359.05	*are theſe* Gage-points *upon* D,	18.95	A. G.
	294.11		17.15	W. G.

To which are ſet the Letters *a. g.* and *w. g.* upon the Rule.

Now, by theſe *Gage-points* the Area of any Circle may be thus found:

For Ale-Gallons.

Set 18.95 (the Gage-point for Ale-Gallons) upon D, to 1 upon C; then againſt any Diameter upon D, you have the Area upon C.

So, if the Diameter were 40 Inches, the Area will be 4.45 Ale-Gallons: And the Rule being thus ſet, the Lines are in effect a Table of Circles Area's; for I likewiſe find, that if the Diameter

be	45	the Area will be	5.64	A. G.
	50		6.16	
	52.5		7.67	
	58.6		9.56	

The like for Wine-Gallons, by the proper Gage-point.

Note, When the Area of any Circle is ſought in Ale-Gallons, if the Diameter be more than 18.95, and leſs than 100, ſet the the Gage-point upon D, to 1, at the Beginning of C; then againſt any Diameter, from the Gage-point to 100, upon D, you have the Area upon C. Thus, the Rule being ſet for Ale-Gallons, you will find that when the

Diameter is	the Area will be Gallons.
30	2.507
40	4.456
60	10.026
80	17.825
100	27.851

When the Diameter is leſs than the Gage-point, or more than 100, then ſet the Gage-point to 1, in the middle upon C; then againſt theſe

Diameters upon D,	you have theſe Area's upon C, A. G.
15	.627
13	.471
10	.278

And without moving the Rule, if 10 at the Beginning of the Line D, be 100, the Area againſt it will be 27.851: So againſt the Diameter, *viz.* 200, the Area is 111.4 Gallons; againſt 300, it is 250.7, and ſo on to 600 Inches Diameter, againſt which you have 1002.6, &c. by this you may further obſerve, that if the Diameter be increaſed by Tens, the Area will increaſe by Hundreds: Thus,

the Diameter	gives Gallons.
10	.27851
100	27.851
60	10.026
600	1002.6

The like for Wine-Gallons by the proper Gage-point.

Laſtly, To find any Part of the Area of a Circle, in Ale or Wine-Gallons.

Set the Gage-point $\frac{1}{3}$, $\frac{1}{2}$, or any other Part of 1; then againſt the Diameter you have the like Part of the Area.

Example. *Let it be required to find the third Part of the Area of a Circle in Ale-Gallons.*

Set the Gage-point to one third of 1, *viz.* .333; then againſt any Diameter you have one third of the Area. Thus, againſt 100 is 9.283, which is one third of 27.85, the whole Area.

Underſtand the like for Wine-Gallons, Ale or Beer-Barrels.

PROB. III. Fig. 2. Plate III.

The two Diameters (c d *and* e f) *of an* Ellipſis *being given, to find the Area or Content in Ale-Gallons.*

As the Square of a Diameter of a Circle is to the Area of that Circle; ſo is the Rectangle or Product of the greater and leſſer Diameters of an Ellipſis to the Area thereof. Therefore multiply the greater Diameter by the leſſer Diameter; then that Product multiplied or divided by the fixed Multiplicators or Diviſors given (in Page 54, and 55.) gives the Area in Inches, Gallons, or Barrels, according to the Number made uſe of.

Or thus, (by Problem VIII. Sect. I.) find a Geometrical Mean Proportion between the greater and leſſer Diameters; for this Mean is the Diameter of a Circle, whoſe Area is equal to the Area of the Ellipſis.

EXAMPLE. Fig. 2. Plate III.

Let the greater Diameter *e d* be 72 Inches, and the leſſer *e f*, 50; by the foregoing Rule the Geometrical Mean between them will be found to be 60 the Diameter of a Circle equal to the Ellipſis, and the Area of a Circle, whoſe Diameter is 60, will be found to be 10.02 Ale-Gallons. But the Area of an Ellipſis may be more eaſily found by the Rule, thus;

Set 359.05 upon B, to one of the Diameters (ſuppoſe 50) upon A; then againſt the other Diameter 72 upon B, you have the Area upon A, which in this Example will be 10.02. Ale-Gallons, the Content of this Ellipſis at one Inch deep: The like may be done for Wine-Gallons, if inſtead of 359.05, you take 294.11.

PROB. IV.

To find the Area or Content of a Triangular Superficies.

I*N all Right-lined Triangles, multiply half the Baſe by the Perpendicular* (which is always the neareſt Diſtance from the Baſe to the oppoſite Angle;) or *multiply half the Perpendicular by the whole Baſe; the Product will be the Area required.*

Example. Let Fig. 3. Plate III. repreſent a Triangular Back or Cooler, whoſe Baſe *a b*, is 260 Inches, and the Perpendicular Line *c o*, 110 Inches, the half *a b* is 130; this multiplied by 110, gives 14300 for the Area in Square Inches, and this divided by 282, quotes 50.7 the Area in Ale-Gallons; or by the Rule at one Operation, thus:

Set 282 upon B, to 130 upon A; then againſt 110 upon B, is 50.7 upon A the Content as before.

To meaſure Quadrangular Figures.

PROB. V.

In any Right-angled Parallelogram, be it Square or Oblong, the Product of any two Sides, including one and the ſame Angle, is equal to the Area or Content.

EXAMPLE I.

IN Fig. 5. Plate III. the Sides *b i* and *i k*, are equal; ſuppoſe each be 128 Inches; this multiplied by it ſelf, is 16044, the Area in Square Inches: For the Area in Gallons by the Rule,

Set 282 upon B, to 138 upon A; then againſt 138 upon B, is 67.53, the Area in Ale-Gallons at one Inch deep.

EXAMPLE II. *Of an Oblong.* (Fig. 4. Plate III.)

Suppoſe *n s*, 130 Inches, and *s y*, 180, the Product of theſe (being multiplied) is 23400, the Area in Inches: But for Ale-Gallons, ſet 282 upon B, to 180 upon A; then againſt 130 upon B, is 32.97 Ale Gallons, the Area required.

PROB. VI.

If any Oblique-angled Parallelogram *(be it a* Rhombus *or* Rhomboides*) multiply the ſhorteſt Diſtance between the two longeſt Sides, by one of the ſaid Sides, the Product is the Area in Inches.*

EXAMPLE I.

Of a Rhombus (Fig. 7. Plate III.)

SUppoſe the Side *p t*, were 130 Inches, and the Diſtance *q y*, 108; theſe multiplied, the Product will be 14040, the Area in Inches; But by the Rule thus:

As 282 is to 108; *So* is 130 to 49.78, the Area in Ale-Gallons at one Inch deep.

EXAMPLE III.

Of a Rhomboides. (Fig. 6. Plate III.)

Let the Side *x z*, be 260, and the Diſtance *t w*, 108; the Product of theſe is 280.80, the Area in Inches. By the Rule:

Set 282 upon A, to 260 upon B; then againſt 108 upon A, is 99.57, the Area in Ale-Gallons.

PROB.

PROB. VII. (Fig. 8. Plate III.)

To find the Area or Content of a Trapezium.

A Right Line drawn from one of the Acute Angles to the Angles oppoſite (as the Line *a b*) will divide the *Trapezium* into two Triangles, the Area's of which are equal to the whole *Trapezium*, and may be found by the 4th Problem of this Section. Or thus: Multiply the Sum of the two Perpendiculars (*f e* and *c r*) by the half of *a b*; the Product is the Area of the *Trapezium*. This is ſo plain, that it needs no Example.

All other irregular Right-lined Figures conſiſting of more than four Sides, muſt be divided into Triangles (which will be ever leſs by 2 than the Number of Sides,) and then the Area's of all thoſe Triangles are equal to the Area of the whole Figure.

PROB. VIII.

The Sides of any Regular Polygon *being given, to find the Area.*

IN any Regular Polygon, multiply half the Sum of the Sides by the Perpendicular (or neareſt Diſtance from the Center to one of the Sides) the Product is the Area.

Example. In a Pentagon (or Figure of five equal Sides) as Fig. 9. Plate III.

Suppoſe each Side be one Inch, to find the Perpendicular (*c a*) we have given *a b* equal to .5, and the Angle at C equal to 36 Degrees.

Therefore,

		L. g.
As the *Sine* of C=36 *d.*	*Co. Ar.*	0 230781
Is to the Side A B=, 5		$\bar{1}$ 698970
So is the *Sine* of B=54 *d.*		9.907957
To the Side C A=.68819		1.837708

which is the Perpendicular ſought; and this .68819 multiplied by half the Sum of the Sides, *viz.* 2.5, the Product is 1.72047, the Area of a Pentagon, whoſe Side is one Inch; and by this Method I find the Area of all the other Polygons, which are expreſt in the following Table. In the firſt Column of which you have the Names of the Polygons; in the ſecond their Areas in Inches; each of which being divided by 282, gives the Area in Ale-Gallons, which are the Numbers in the third Column; and in the 4*th* and 5*th* Columns you have the reſpective Diviſors.

Names of the Polygons.	Multiplicators. *Being the Area's in Inches and Gallons, when the Side is one Inch.*		Diviſors. *Being the Squares of the Sides, when the Area is one Inch or one Gallon.*	
	Inches.	Ale-Gall.	Inches.	Ale-Gall.
Trigon	.433013	.001535	2,309401	651.251
Tetragon.	1.000000	.0035461	1.000000	282.000
Pentagon	1.720477	.0061010	.581234	163.908
Hexagon	2.598076	.0092130	.384000	108.542
Heptagon	3.633959	.0128864	.275182	77.601
Octagon	4.828427	.0171221	.207107	58.404
Nonagon	6.181824	.0219214	.161764	45.615
Decagon	7.694209	.0272844	.129968	36.051

Now having the Area (both in Inches and Ale-Gallons) of theſe Polygons when the Side is Unity, the Area of each may be readily found, the Side being given: For,

As the Square of the Side of any Polygon, is to the Area of that Polygon;
So is the Square of the Side of any other Polygon, to the Area thereof: Therefore,

If the Area of any Polygon (expreſſed in the Table) be deſired, in Inches or Ale-Gallons.

Multiply or divide the Square of the Side by the reſpective Multiplicator or Diviſor, the Product or Quotient will be the Area ſought.

Example: For the Area in Inches, ſuppoſe the Side of a Pentagon be 50 Inches; the Square is 2500. This multiplied by 1.720477, or divided by .581234, gives 4301 2, the Area in Inches.

For the Area in Ale-Gallons, if the Square of 50 (*viz.* 2500) be multiplied by .006101, or divided by 163.908, the Product or Quotient will be 15.252, the Area ſought.

To find the Side by the Area given.

Multiply or divide the given Area by the proper Multiplicator or Diviſor; the Product or Quotient will be the Square of the Side, whoſe Square Root is the Side ſought.

Thus, the Area of a *Pentagon* is 15.252 Ale-Gallons. This multiplied by 163.908, or divided by .006101, gives 2500, whoſe Square Root is 50, the Side ſought.

But the Lines C and D, will effect this with much more Expedition; for at once ſetting of the Rule, you have the Area (in Inches or Ale-Gallons) to any given Side, or the Side to any Area given.

Example. For the Area in Ale-Gallons, ſet 1 upon D, to (the Area in Gallons when the Side is 1, *viz.*) .006101, upon C; then againſt any Side upon D, you have the Area upon C; and the contrary.

Thus, againſt 50 upon D, is 15.252, the Area in Ale-Gallons; alſo againſt 22 Gallons, an Area upon C, you have 60, the Side upon D, &c.

SECT. IV.

The Uſe of the Rule in the Menſuration of Solids; and firſt, of a Priſm.

A *Priſm* is a Solid bounded by ſeveral *Planes*, two of which being oppoſite, are called *Baſes*; and theſe are equal, parallel, alike, and alike ſituate; but the other *Planes* are Parallelograms, in which a Right Line may be every where applied from Baſe to Baſe. Under this Name *Priſm* is comprehended that Solid of two Circular Baſes, uſually called a *Cylinder*.

PROB. I.

To find the Solid Content of a Priſm.

MUltiply the Area of the Baſe by the Perpendicular Height, the Product is the Content ſought.

Example. *Let* Fig. 11. Plate III. *repreſent a Brewer's* Tun, *in the Form of a ſquare Priſm, whoſe Baſe* g h k i, *is equal to* Fig. 5. *each Side being* 138 *Inches, and the Perpendicular* (s i) 30 *Inches. How many Ale-Gallons or Beer-Barrels will this Tun contain?*

Set 282 upon B, to 138 upon A; then againſt 138 upon B, is 67.534 upon A, the Content in Ale-Gallons at one Inch deep; this multiplied by 30, the whole Depth, gives 2026.02, the whole Content in Ale Gallons; this divided by 36, quotes 56.278, the Content in Beer-Barrels.

The Fraction .278 (being reduced) is 10 Gallons, and .008 Parts. For,

As 1.000 : 36 :: .278 : 10.008.

2. Example. *Of a Round Priſm or Cylinder.*

Let Fig. 12. Plate III. repreſent a Round Tun, whoſe Diameter (*n o*) at Top is equal to (*s y*) the Diameter at Bottom, each being 120 Inches, and the Altitude (*e x*) 36 Inches: How many Gallons or Beer-Barrels may this contain? Firſt, for Gallons.

Set the Gage-point (*a g*) to 36, the Tun's Depth upon C; then againſt 120, the Diameter, upon D, is 1443.6, the Content in Ale-Gallons.

2. For the Content in Beer-Barrels: 1443.6 divided by 36, quotes 40.1, the Content in Barrels.

Suppoſe I come to this Tun, and find the Liquor 9 Inches deep: How many Gallons are contained in the Tun?

Set the Gage-point to 9; then againſt 120 is 160.95 Gallons.

PROB-

PROB. II.

The Diameter, Depth and Content of any Cylindrical Tun, *any two being given to find the third.*

IN this Problem are three Queſtions, all reſolved at once ſetting the Rule.

1. By the the Depth and Content, to find the Diameter.

Example. *Suppoſe the Depth* 40 *Inches, and the Content* 1800 *Ale Gallons. What is the Diameter?*

Set 40, the Depth upon C, to the Gage-point upon D; then againſt 1800, the Content upon C, is 127.1 Inches, the Diameter ſought.

2. By the Diameter and Content, to find the Depth, without moving the Rule: Say,

As 127.1, the Diameter	upon	D
is to 1800, the Content;		C
So is the Gage-point		D
to 40, the Depth,		C

3. By the Depth and Diameter to find the Content, the Rule ſtanding as before: Say,

As the Gage-point	upon	D
is to 40, the Depth;		C
So is 127.1, the Diameter,		D
to 1800, the Content,		C

Note, As a Circle is the Baſe of a Cylinder or round Priſm; ſo a Triangle, Quadrangle, or any other plain Superficies, may repreſent the Baſe of a Priſm: For if there be Planes erected perpendicularly upon the Lines which encompaſs any ſuch Superficies, they will generate a Solid, which may be called a *Priſm*; and the Content of any ſuch Solid is found by multiplying the Area of the Baſe by the Altitude, or Diſtance from one Baſe to another.

SECT. V.

The Uſe of the RULE in the Menſuration of a Sphere, and its Fruſtums.

The Diameter of a Sphere or Globe being given in Inches, to find the Content thereof in Inches or Gallons, both Ale and Wine.

A Sphere is two third Parts of a Cylinder, whoſe Diameter and Altitude are equal to the Diameter of the Sphere.

But if the Diameter and Altitude of a Cylinder be 1, the Area of the Baſe, which is alſo the Solid Content (for 1 doth not multiply) is .785398: Therefore $\frac{2}{3}$ of .785398, *viz.* .523598, is the Content of a Sphere, whoſe Diameter is Unity, or 1.

Moreover, all Bodies are in Proportion one to another, as the Cube of their like Sides. Therefore,

As the Cube of the Diameter of any Sphere, *Is* to the Content of that Sphere ;
So is the Cube of the Diameter of any other Sphere, *To* the Content thereof.

Now, if the Diameter be 1, the Cube thereof is but 1. Therefore,

As 1 is to .523598 ;
So is the Cube of the Diameter of any Sphere, to the Content.

Example. *Let the Diameter of a Sphere be* 20 *Inches, and the Content required in ſolid Inches.*

By the Lines D and C, on the Rule, the Cube of 20 will appear to be 8000. This multiplied by .523598, is 4188.784, the Content ſought.

For the Content in Gallons.

If .52358 be divided

By	282	gives	.0018567	the Content in	A. G.
	231		.002266		W. G.

Now, if the Cube of the Diameter of any Sphere be multiplied by either of theſe Numbers, the Product will be the Content in Ale or Wine-Gallons reſpectively. Thus, the Cube of 20, the Diameter (*viz.*) 8000, multiplied by .001856, is 14.848, the Content in Ale-Gallons ; and the like may be done for Wine.

But the Content of any Sphere may be more readily found by the Inſtrument. Thus,

Set 1 upon D, to .523598 upon E; then againſt any Diameter upon D, is the Content upon E; So againſt 20 (the Diameter above-mentioned) upon D, is 4188.784, the Content in Inches, as before. In like manner, if the Diameter were 30, the Content will be 14137.14, *&c.*

For Ale-Gallons *thus:*

Set 1 upon D, to .001856 upon E; then againſt any Diameter upon D, is the Content upon E; as againſt 20 upon D, is 14.84, the Content as before; and without moving the Rule, I find, that if the

$$\text{Diameter be}\left\{\begin{matrix}40\\50\end{matrix}\right\}\text{the Content is}\left\{\begin{matrix}50.13\\118.38\end{matrix}\right\}\text{A. G.}$$

PROB. II.

Having the Altitude of the Fruſtum *of a Globe, together with the Diameter of its Baſe, to find the Altitude of its other* Fruſtum; *or (which is all one) the Remainder of the* Globe's *Axis.*

The RULE is,

DIvide the Square of the Semidiameter of the *Fruſtum*'s Baſe, by the Altitude of either of the *Fruſtums*, the Quotient will be the Altitude of the other *Fruſtum*.

Example. In Fig. 10. Plate III. let *c b*, the Altitude of the *Fruſtum*, be 6 Inches, and *d c e*, the Diameter at the Baſe, 24; the half of this, (*viz.* *d c*) is 12; which ſquared, is 144; this divided by the Altitude *b c*, *viz.* 6, the Quotient is 24, equal to *c a*, the Altitude of the other *Fruſtum*.

Note alſo, That a Geometrical Mean Proportion between *a c* and *b c*, is equal to *c d*; which doubled is *d e*, the Diameter of the *Fruſtum*'s Baſe. *Euclid* 6, 13.

PROB. III.

Having the Altitude of the leſſer Fruſtum *of a* Sphere, *and the Diameter of the Baſe, to find the Content in* Inches *or* Gallons.

MUltiply the Square of half the Diameter of the Baſe by three times the Altitude; and to this Product add the Cube of the Altitude, the Sum multiply'd

By	.523598	Gives the Content in	Inches.
	.0018567		Ale-Gallons.

Example. In Fig. 10. Plate III. let *d c e* the Diameter of the leſſer *Fruſtum* (*c d b e*) be 24 Inches, and *c d* its Altitude, 6 Inches. What is the Content in ſquare Inches?

e e ſquared =	144
3 times *b c* =	18
	1152
	144
	2592
Cube of *b c* =	216
	2808

2808 multiplied

			Content.
By	.523598	gives	1470.263 Inch.
	.0018567		5.2137 Ale-Gall.

The Content of the leſſer *Fruſtum* taken from the Content of the whole Sphere, leaves the Content of the greater *Fruſtum.*

SECT. VI.

The Uſe of the Inſtrument *in the Menſuration of* Pyramids *and their* Fruſtums.

A Pyramid *is a Body contained under ſeveral Planes, ſet upon one right-lined* Baſe, *and meeting in a Point at the Top, which is called the* Vertex; *alſo in each of thoſe Planes a Right Line may be every where applied from the* Baſe *to the* Vertex.

And under this Name *Pyramid* is comprehended that pyramidical Body (whoſe Baſe, is a Circle) commonly called a *Cone.*

PROB. I.

To find the Content of a Pyramid.

A *Pyramid* is one third Part of a Priſm, which hath the ſame Baſe and Altitude. Therefore,

Multiply the Area of the Baſe by $\frac{1}{3}$ of the Altitude, or $\frac{1}{3}$ of the Area by the whole Altitude; in either Caſes the Product will be the Content of the Pyramid.

EXAMPLE I.

In a ſquare Pyramid (as Fig. 17. Plate III.) ſuppoſe the Side of the Baſe *xp*, be 112 Inches, and the Altitude *zs*, 90 Inches: What will be the Content in Ale-Gallons?

Set 282—— } upon { to 112 } upon
Then againſt 112 } B { is 44.482 } A,

the Area of the Baſe in Ale-Gallons. Then ſet 1, upon A, to 30 upon B; and againſt 44.482 upon A, is 1334.46, the Content in Ale-Gallons.

EXAMPLE II.

There is a round Pyramid or Cone (as Fig. 18. Plate III.) ſuppoſe the Diameter at the Baſe *ano*, be 100 Inches, and the Altitude *nx*, 180: How many Ale-Gallons may this Cone contain?

Set the Gage-point for an Ale-Gallon, to $\frac{1}{3}$ of the Altitude, *viz.* 60, upon C; then againſt 100 upon D, is 1671, the Content required.

PROB. II.

To find the ſolid Content of the Fruſtum *of a* Pyramid.

1. *BY the Rules aforegoing, find the Area of the greater and alſo of the leſſer Baſe.*

2. *Find a Geometrical Mean Proportional betwixt theſe two Area's.*

3. *The Sum of theſe three Numbers being multiplied by one third Part of the Altitude, the Product is the Content of the* Fruſtum *ſought.*

Example. Let *a e e o,* (in Fig. 18. Plate III.) repreſent the Fruſtum of a round Pyramid or Cone, *a o,* the Diameter of the greater Baſe is 108 Inches, *c e,* the Diameter of the leſſer Baſe, 81, and *n n* the Altitude 36 Inches, to find the Content in Ale-Gallons.

1. Set the Gage-point upon 1, at the Beginning of the Line C; then againſt 108 upon D, is 32.48, the Area of the greater Baſe in Ale-Gallons.

2. Set the Gage-point to 1, at the Beginning of the Line C; then againſt 81 upon D, is 28.27, the Area of the leſſer Baſe.

3. Set 32.48 upon C, to 32.48 upon D; then againſt 18 27 upon C, is 24.36, the Geometrical Mean between the two Areas.

So the Area of the greater Baſe is ———— 32.48
The Area of the leſſer is ———— 18.27
A Geometrical Mean betwixt the 2 Area's is — 24.36

The Sum is ———— 75.11

This multiplied by one third of the Altitude, *viz.* 12, gives 901.32, the Content of the Fruſtum ſought.

Or by the Pen thus:

1. Square the Diameter of the greater Baſe.
2. Square the Diameter of the leſſer Baſe.
3. Multiply one of theſe Diameters by the other. To this Product add the two Squares, and multiply the Sum by the Altitude: The laſt Product, divided by 1077, gives the Content in Ale-Gallons.

SECT. VII.

The Uſe of the Rule in Gauging of Brewer's Tuns.

THESE Veſſels are in ſeveral Forms: The moſt uſual may be conſidered under ſome of theſe Varieties:

I. *Such whoſe* Baſes *are equal.*

And if the equal Baſes are alike (that is, both Round, both Square, *&c.*) and Parallel, and the Sides of the Tun ſtraight from one Baſe to the other, theſe Tuns are called *Priſms.* See Problem I. of this Section.

II. *Such*

II. *Such whoſe* Baſes *are unequal.*

1. If the unequal Baſes be alike, (ſuppoſe both Rectangular Parallelograms, both Elliptical, *&c.*) parallel, proportional, and alike ſituate, and the Sides of the Tun ſtraight, ſuch Tuns are the *Fruſtums* of Pyramids. See Prob. II. of this Section.

2. If the unequal Baſes be unlike, ſuppoſe one Round, and the other Elliptical; or if they be alike but not proportional; or if they be both alike and proportional; yet if they be not alike ſituate, the Tun is called a *Priſmoid*, in which 'tis ſtill ſuppoſed the Baſes are parallel, and Sides of the Tun ſtraight. See Prob. VII.

Note, When the Baſes are proportional, it will be,

As the Length of the greater Baſe, is to the Length of the leſſer:
So is the Breadth of the greater Baſe, to the Breadth of the leſſer.

And the Baſes are ſaid to be alike ſituate, when the Length of the greater and leſſer Baſes are both in the ſame Plane.

PROB. I. Fig. 11, Plate III.

There is a Tun, whoſe Baſes are both equal, parallel, and ſquare, each Side of either Baſe being 138 *Inches, and the Depth* 32 *Inches: What is the Content of this Tun in Ale-Gallons?* Anſwer 2228.622. *For,*

SET 282 upon B, to 138 upon A; then againſt 138 upon B, you have 67.534 upon A, the Content at one Inch deep; this multiplied by the whole Depth, *viz.* 33, gives 2228.622, the Content required. Or thus:

Set 16.79 upon D, to 33 (the Tun's depth) upon C; then againſt 138 upon D, is 2228.622 upon C, the Content as before.

Note, 16.79, is the Gage-point for ſquare Veſſels, it being the Square Root of 282, the Inches in an Ale-Gallon.

It is needleſs to give more Examples of this Sort of Tuns: For let the Baſes be in any Form whatſoever, if the Tun have the Qualifications above-mentioned, the Content is found by multiplying the Area of the Baſe by the Depth; and if the Baſes are round, as in moſt Tuns they are, the Content may be found at one Operation, as hath been ſhewn in the Firſt and Second Problem of Sect. IV.

Of Tuns, whoſe Baſes are unequal.

PROB. II. Fig. 16. Plate III.

There is a Tun (*as* a b c d e f g h) *whoſe Baſes are rectangular Parallelograms, unequal, but parallel, and alike ſituate, and the Sides proportional*; (d a) *the Length of the greater Baſe is* 100 *Inches, and* (d c) *the Breadth* 80; (h e) *the Length of the leſſer Baſe* 85, *and* (h g) *the Breadth* 58, *and the Depth of the Tun* 30 *Inches. How many Gallons may this Tun contain?*

FOR all Tuns of this Sort, this is a general Rule:

Take the Area of the greater Baſe, the Area of the leſſer Baſe, and a Geometrical Mean Proportional betwixt the two Area's; the Sum of theſe three multiplied by one third part of the Tun's Depth, gives the Content.

Example either by the Pen, or the Lines A and B upon the Rule.

As 282 is to *d a*, 100; *So* is *d e*, 80 to the Area of the greater Baſe, *viz.*	28.368
As 282 is to *b e*, 85; *So* is *b g*, 68 to the Area of the leſſer Baſe,	20.496
A Geometrical Mean between theſe two Areas will be found by Problem VIII. Sect I. to be —— ——	24.112
The Sum is —— ——	72.976

This multiplied by one third Part of the Tun's Depth, *viz.* 10, gives 729.76, the Content in Ale-Gallons.

PROB. III. Fig. 16. Plate Plate III.

There is a Tun, whoſe Baſes (i l k m *the greater, and* n q o p *the leſſer*) *are both Elliptical, Parallel, Proportional, and alike ſituate; the greater and leſſer Diameter of each Baſe being equal to the Lengths and Breadths of the Baſes of the former Tnn, and the Depth the ſame. How many Gallons will this Tun contain?*

THIS Elliptical Tun may be conceived to be inſcribed in the Tun, whoſe Content was found in the laſt Problem: All the given Dimenſions are the ſame in both, and the Operation will be the ſame too: For the firſt Term in the Proportion we take 359.05, inſtead of 282. See the Work.

As 159.05 is to *i k*, 100 ; *So* is *l m*, 80 to the Area of the greater Baſe, ——— } 22.281

As 359.05 is to *n o*, 85 ; *So* is *p q*, 68 to the Area of the leſſer Baſe, ——— } 16.093

The Geometrical Mean between thoſe two Area's will be found by Prob. VIII. Sect. I. to be ——— ——— } 18.938

The Sum is ——— ——— ——— 57.317

This multiplied by one third Part of the Depth, *viz.* 10, gives 573.17, the Content of this Elliptical Tun, in Ale-Gallons.

PROB. IV.

There is a Tun (*as* Fig. 17. Plate III.) *whoſe Baſes are both ſquare, but unequal* (x p) *the Side of the greater Baſe is* 108 *Inches,* (r o) *the Side of the leſſer Baſe* 93, *and the Depth* (s a) 30 *Inches. What is the Content in Ale-Gallons?*

THIS Tun being the Fruſtum of a Pyramid, may be gauged by the general Rule in Problem II. of this Section. Or thus:

1. Square the Side of the greater Baſe, and alſo the Side of the leſſer Baſe.

2. Multiply the greater Side by the leſſer, and to the Product add the two former Squares.

3. The Sum of theſe three being multiplied by one third of the Depth, gives the Content in Square Inches, which divided by 282, the Quotient is the Content in Ale-Gallons.

Example. By the Inſtrument the Square of *x p*, 108 is —— —— —— 11664

The Square of *o r*, 93, is —— —— 8649
108 multiplied by 93, is —— —— 10044

Sum is —— —— —— 30357
Multiplied by *one third* Altitude —— 10

The Product, *viz.* —— —— 303570

This divided by 282, quotes 1076.48, the Content ſought.

PROB.

PROB. V. Fig. 18. Plate III.

There is a Conical Tun, whoſe Baſes are both Circular, (a o) *the Diameter of the greater Baſe is* 108 *Inches,* (c e) *the Diameter of the leſſer Baſe* 93, *and* (u n) *the Depth* 30 *Inches: What is the Content of this Tun in Ale-Gallons?*

THE Dimenſions being the ſame as in Problem IV. of this Section, proceed as is there directed, the laſt Product will be 303570; this divided by 359.05, quotes 845.48, the Content ſought.

Or thus by the Rule.

Subtract the Diameter of the leſſer Baſe from the Diameter of the greater, and add half the Difference to the leſſer, the Sum is the Diameter in the Middle of the Tun's Depth; which found,

1. Set the Gage-point (*a g*) to the Tun's Depth upon C.

Then againſt the Diameter upon D, is a fourth Number upon C, which keep.

2. Set the Gage-point to $\frac{1}{3}$ of the Tun's Depth upon C.

Then againſt $\frac{1}{2}$ the Difference of the Diameters upon D, is a fourth Number; which added to the fourth Number firſt found, is the Content of the Tun ſought.

EXAMPLE. Fig. 11. Plate III.

a o the Diameter of the greater Baſe is —— 108
c e the Diameter of the leſſer Baſe is —— 93

The Difference is —— —— —— 15

The half Difference is 7.5; this added to 93, makes 100.5 for the Diameter in the Middle of the Tun's Depth: Therefore,

1. Set the Gage-point to 30, the Tun's Depth; then againſt 100.5, the Diameter, is 843.91, which keep.

2. Set the Gage-point to half of the Tun's Depth, that is, 10; then againſt 7.5, the Semi-difference of the Diameters, is 1.56. This added to 843.91 (the Number firſt found) the Sum is 845.47, the Content ſought: Which agrees very well with the former Rule.

PROB. IV.

To find the Content of a Pyramidical Tun, whoſe Baſes are in the Form of any of the firſt 10 *Regular Polygons, one Side of each Baſe and the Tun's Depth being given.*

THE Area of each Baſe of any ſuch Tun may be found by Prob. VIII. Sect. I.

A Geometrical Mean betwixt theſe Areas may be found by Prob. VIII. Sect. I.

The Sum of the two Area's and the Geometrical Mean between them, being multiplied by half of the Tun's Depth, gives the Content. Or thus:

By the Rule in the laſt Problem ſquare the Side of each Baſe; multiply the greater Side by the leſſer; then multiply the Sum of theſe Three by half the Tun's Depth. Laſtly, multiply or divide this laſt Product by the proper Multiplicator or Diviſor [as *per* Table, page 63] the Product or Quotient will be the Content in Ale Gallons.

Example. Let there be a pyramidical Tun, whoſe Baſes are both in form of a Pentagon (or Figure of five equal Sides,) and let the Side of the greater Baſe be 108, the Side of the leſſer 93, and the Depth 30 Inches; all the ſame as in the laſt Problem: Therefore, according to the Rule and Example there laid down, the laſt Product will be 303770: This multiplied by .006101, or divided by 163.908, [*Vid.* Table, Page 63] gives 1852.08.

To find the Content of any Tun, whoſe parallel Baſes are Rectangular Parallelograms, or Ellipſes, the Sides of the Tun being ſtraight from the Top to the Bottom, whether the Baſes be alike or unlike, proportional or diſproportional, alike ſituated or inverted, by one general Rule.

PROB.

PROB. VII.

Let Fig. 16. Plate III. *repreſent a Tun called a Priſmoid, the Baſes are Rectangular Parallelograms, unequal and diſproportional, but parallel and alike ſituate*; (a d) *the Length below* = 150 *Inches, and* (a b) *the Breadth* 100, (e h) *the Length above* 90, *and* (e f) *the Breadth* 70, *and the Depth* 30. *What is the Content in Ale-Gallons?*

RULE.

1. TO the greater Length (*a d*) add half the leſſer Length (*e b*,) and multiply the Sum by the greater Breadth (*a b*,) reſerving that Product.

2. To the leſſer Length (*e b*) add half the greater Length (*a d*) and multiply the Sum by the leſſer Breadth (*e f*) add this Product to the former reſerved Product.

3. Multiply the Sum of theſe two Products by half of the Tun's Depth, and divide the laſt Product by 282, the Quotient will be the Content of the Tun in Ale-Gallons.

Example. (*a d*) 150 added to half (*e b*) 45, the Sum is 195; this multiplied by (*a b*) 100, the Product is 19500, which keep.

2. (*e b*) 90 added to half (*a d*) 75, the Sum is 165; this multiplied by (*e f*) 70, the Product is 11550; this added to the former Product, *viz*, 19500, the

Sum is 31050; this multiplied by half of the Depth, *viz.* 10, the Product is 310500; and this divided by 282, gives 1101.06, the Content in Ale-Gallons.

The Truth of this Rule will plainly appear, if we duly conſider Fig. 13. in which (*a b c d*) is equal to the greater Baſe, and (*e f g b*) equal to the leſſer Baſe of the Tun in the laſt Problem (which was repreſented by Fig. 16.)

In this Figure 13, (*b i e v*) is made equal to (*f e g b*).

Alſo (*a b*) 100, leſs (*e f*) 70, is equal to (*a i*) 30.

And (*a b*) 150 leſs by (*e b*) 90, is equal to (*s d*) 60, the Altitude is 30, as before.

The Lines being drawn, the whole Solid is compoſed of theſe Parts, *viz.* three Priſms and a Pyramid.

$$\text{The}\left\{\begin{array}{c}1\\2\\3\end{array}\right\}\text{Priſm is}\left\{\begin{array}{l}b\ i\ o\ v\ f\ e\ b\ g\\i\ a\ s\ o\ b\ e\\o\ v\ c\ n\ b\ g\end{array}\right\}$$

The Pyramid is *o s n d b*.

The Content of theſe four Solids being added together, gives the Content of the whole Solid *a b c d e f g b*; for the whole is equal to all its Parts taken together.

And the Content of theſe may be found thus:

1. Multiply (*f g*) 90 by (*f e*) 70, the Product is 6300; this multiplied by (*b f*) the Altitude, *viz.* 30; gives 189000, the Content of the Priſm, *b i o v f e b g*.

2. Multiply (*c i*) 30 by (*a s*) 90, the Product is, 2700; this multiplied by half the Altitude, *viz.* 15, is 40500, the Content of the Priſm *i a s o b e.*

3. Multiply (*e n*) 70 by (*n o*) 60, the Product is 4200; this multiplied by 15, is 63000, the Content of the Priſm *o v c n b g.*

4. Multiply (*s d*) 60 by (*s o*) 30, the Product is 1800; this multiplied by one third of the Altitude, *viz.* 10. gives 18000, the Content of the Pyramid *s o n d b.*

So the Content of the	1	Priſm is	189000
	2		40500
	3		63000
Pyramid is			18000
The Sum			310500

Is the Content in ſquare Inches, and is exactly the ſame with the Content found by the Rule before given.

The Truth of this Rule being thus demonſtrated, we ſhall ſhew its further Uſefulneſs in the following Problems.

PROB.

PROB. VIII. Fig. 16. Plate III.

There is a Tun (called a Cylindroid) *whoſe Baſes are both elliptical, unequal, and diſproportional; (that is, there is not the ſame Proportion betwixt the Axis and Diameter of the greater Baſe, as is betwixt the Axis and Diameter of the leſſer) let the given Dimenſions be the ſame as in the laſt Problem,* viz.

$$\left.\begin{array}{l} no=90 : qp=70 \\ ik=150 : lm=100 \end{array}\right\} \begin{array}{l} \text{Rectangular} \\ \text{Conjugates} \end{array} \left\{ \begin{array}{l} \text{above.} \\ \text{below.} \end{array}\right.$$

And the Depth 30 Inches: What is the Content of this Tun in Ale Gallons?

For the Content of this Tun work by the General Rule, and according to the Example given in the laſt Problem; only inſtead of 282, divide the laſt Product by 359.05. So in this Example (the Numbers being the ſame as in the laſt) the laſt Product will be 310500; this divided by 359.05, gives 864.78, the Content in Ale-Gallons.

PROB. IX. Fig. 15. Plate III.

There is a Tun, whoſe Baſes are Rectangular Parallelograms, and Parallel, but neither equal nor alike ſituate (for g h, *the Breadth above, is poſited directly oppoſite to* a d, *the Length below:) How many Ale-Gallons will this Tun contain?*

The Dimenſions are,
b g Breadth = 68 : *b e* Length = 85 above.
d a Length = 100 : *d c* Breadth = 80 below.
The Depth 30 Inches.

If the Baſes of this Tun were alike ſituate (as is ſuppoſed in Problem II. of this Section) the Content would be 729.76 Ale-Gallons, as was there ſhewn; But the Baſes being inverted, as above ſaid, the Content will be found to be 735.81 Ale-Gallons, as appears by the following Work, which is performed by the General Rule laid down in Problem VII. of this Section.

Thus (*b g*) 68 added to half (*d a*) 50, is 118; this multiplied by (*b e*) 85, the Product is 10030.

Again (*d a*) 100 added to half (*b g*) 34, is 134; this multiplied by (*d c*) 80, the Product is 10720, the Sum of theſe two Products is 20750; which multiplied by one third of the Depth, *viz.* 10, gives 207500, the Content in ſquare Inches; this divided by 282, the Quotient is 735.81, the Content in Ale-Gallons. Or if the Sum of the laſt Products be divided by 359.05, the Quotient will be 577.91, the Content of an Elliptical Tun, whoſe Axis above is directly oppoſite to the ſhorter Diameter below; and ſuch an Elliptical Tun may be conceived to be inſcribed in the other, the reſpective Axis's and Diameters, Lengths and Breadths, and alſo the Depth being the ſame in both.

PROB. X. Fig. 14. Plate III.

To find the Content of a Tun, whoſe Baſes are unlike, ſuppoſe one ſquare, and the other oblong; or one round, and the other elliptical, but Parallel one to the other, and the Sides of the Tun ſtraight.

The Dimenſions are,
Breadth Length
$ef = 74 : fg = 100$ } Rectangular { above.
$dc = 100 : cb = 100$ } Conjugates { below.
Depth 30 Inches.

By the general Rule in Prob. VII. work thus:

1. To (ef) 74 add half (dc) 50, the Sum is 124 this multiplied by fg 100, is 12400.

2. To (dc) 100 add half (ef) 37, the Sum is 137; this multiplied by (ed) 100, is 13700; this added to the former Product, *viz.* 12400, makes 26100; and this multiplied by one third of the Depth, *viz.* 10, the Product is 261000; this divided by 282, gives 925.53, the Content of this Tun in Ale-Gallons, ſuppoſing the Baſes are one Square, and the other Oblong, as the Figure repreſents them.

But if the Baſes were one Round, and the other Elliptical, the laſt Product, *viz.* 261000, muſt be divided by 359.05, and in this Caſe the Content will be 726.91.

By what hath been ſaid, I preſume, it will not be difficult to find the whole Content of any Tun, whoſe Baſes are parallel, and Sides ſtraight from Top to Bottom.

SECT. VIII.

Shewing how to find the Content of any Tun from Inch to Inch, or what any Tun will contain upon every Inch of its Depth, which is commonly called *Inching of a* Tun.

PROB. I.

Suppoſe a Tun, whoſe Baſes are both Round, or both Square, and Parallel one to another, and in the Sides a ſtraight Line may be every where applied from Baſe to Baſe. Let the Diameter, or Side of the greater Baſe be 187.2 *Inches, the Diameter, or Side of the leſſer Baſe* 180, *and the Depth* 12 *Inches, to inch this Tun (be it Round or Square) from the leſſer Baſe downward.*

RULE.

FROM the Side or Diameter of the greater, ſubtract the Side or Diameter of the leſſer Baſe, divide the Difference by the Tun's Depth, and call the (Qotient *a*).

For the Side or Diameter of the leſſer Baſe put (*b*).

Multiply the Number (*b*) by the Number (*a*) and call the Product (*c*). Then,

1. To the Square of (*b*) add the Number (*c*), and one third Part of the Square of (*a*), the Sum of theſe three will ſhew the Content of the firſt Inch of the Tun.

2. Multiply one third Part of the Square of (*a*) by 2; to this Product add the Number (*c*); then multiply the Sum by 2, and to this Product add the Square of (*b*); multiply this Sum by 2, this laſt Product is the Content of the two firſt Inches of the Tun.

3. Multiply one third Part of the Square of (*a*) by 3; then add and multiply as in the laſt, the third and laſt Product will ſhew the Content of the three firſt Inches of the Tun.

4. Having found theſe three Numbers, ſubtract the firſt from the ſecond, and likewiſe the ſecond from the third; then the firſt Number, and each of theſe Differences or Remainders being ſeverally divided by 282 for ſquare Tuns, or by 359.05 for round Tuns, the reſpective Quotients will be the Content (in Ale-Gallons) of the firſt, ſecond or third Inches of the Tun; by which a Table may be made of the whole Tun, as in the Example following.

Example of a Square Tun.

The Side of the greater Baſe ———————— 187.2
The Side of the leſſer Baſe ———————— 180.0

The Remainder is ———————— 7.2

This divided by the Tun's Depth, *viz.* 12, gives .6 for the Number (*a*).

The Number *b* (which is the Side of the leſſer Baſe) is 180; this multiplied by the Number *a*, *viz.* .6, gives 108 for the Number *c*.

1. The Square of *b* is	32400
Number *c* is	108
$\frac{1}{3}$ of the Square of *a* is	.12
The Sum is the Content of the firſt Inch of the Tun, in Inches, *viz.*	32508.12

2. $\frac{1}{3}$ of the Square of *a* is	.12
Multiply by	2
Product	.24
Add the Number *c*	108.00
Sum is	108.24
Multiply by	2
Product	216.48
Add the Square of the Number *b*	32400.00
Sum is	32616.48
Multiply by	2
Product is the Content of the two firſt Inches of the Tun, in Inches, *viz.*	65232.96

3. One third of the Square of *a*	.12
Multiply by	3
Product	.36
Add the Number *c*	108.00
Sum is	108.36
Multiply by	3
Product	325.08
Add the Square *b*	32400.00
Sum is	32725.08
Multiply by	3
Product is the Content of the three firſt Inches of the Tun, in Inches,	98175.24
Two firſt Inches	65232.96
Firſt Inch	32508.12
Reſt for ſecond Inch	32724.84
Three firſt Inches	98175.24
Two firſt Inches	65232.96
Reſt for third Inch	32942.28

Now theſe three Numbers,

viz.	32508.12	Divided	115.277
	32724.84	by 282,	116.045
	32942.28	quotes	116.816

For the Content (in Ale-Gallons) upon the firſt, ſecond, and third Inches of this Tun, ſuppoſing the Baſes ſquare,

And if the ſaid Numbers be divided by 359.05, the reſpective Quotients will

be	90.539 91.142 91.747	the Content of the firſt, ſecond and third Inches of the Tun, when its Baſes are round.

Now by theſe Numbers the following Table is compoſed for the round Tun. Thus,

Take the firſt Inch from the Second, the Remainder or Difference is .603, which ſet down in the third Column, againſt the Space betwixt the firſt and ſecond Inches. In like manner take the ſecond Inch from the third, the Remainder is .605, which place in the third Column under the other; then take .603 from .605, there remains .002 for the ſecond Difference, which is always the ſame, as in the fourth Column; this ſecond Difference added to the ſecond Number in third Column, *viz.* .605, makes .607, and this added to the Content of the third Inch, *viz.* 91.747, makes 92.354, the Content of the fourth Inch; and ſo by a continual Addition of the ſecond Difference to the firſt, and of their Sum to the Inch laſt found, the Table is made.

Depth.

Depth. *Inches.*	*Contents of every Inch in Ale Gall.*	*First* *Differ.*	*Second* *Differ.*
1	90.539		.002
2	91.142	.603	.002
3	91.747	.605	.002
4	92.354	.607	.002
5	92.963	.609	.002
6	93.574	.611	.002
7	94.187	.613	.002
8	94.802	.615	.002
9	95.419	.617	.002
10	96.038	.619	.002
11	96 659	.621	.002
12	97 282	.623	.002
Sum—1126.706-Content of the Tun.			

In this Example we have made a Table for the Round Tun, and the Operation would have been the ſame for the Square Tun, had we taken the three firſt Inches of that inſtead of theſe.

Note, When we inch a Tun by this Rule, we ſuppoſe it to be Regular, having all the Qualifications expreſt in the Problem. But foraſmuch as moſt Tuns have ſome Irregularity or Unevenneſs in their Sides. I ſhall here lay down a more Practical Rule for inching of Tuns.

PROB. II.

There is a Tun whoſe Diameter at the Bottom is 108 *Inches, the Diameter at the Top* 93, *and the Depth* 30; *how many Ale-Gallons will this Tun contain upon every Inch of the Depth?*

TO reſolve this Queſtion, there muſt firſt be known the whole Content of the Tun, and alſo the Area of the mean Diameter in the middle of every 10 or 12 Inches of the Depth, and theſe may be thus found:

Take (with ſome convenient Inſtrument) a Diameter in the middle of every 10 Inches of the Tun's Depth, and ſuppoſe in this Example the firſt (at 5 Inches from the Top) be 95.5, the ſecond (at 15 Inches from the Top) 100.5, and the third 105.5; theſe I ſet down as in the following Table.

This done, we may conſider the Tun either as Round or Square, and firſt to Inch it as a Round.

Set the *Gage-Point* (*a g*) to 1; then againſt the ſeveral Diameters already found, you have their reſpective Area's: So the firſt Area will be 25.40, the ſecond 28.13, the third 31.00; theſe I ſet down in the Table againſt their Diameter, and having added them together, the Sum is 84,53, this multiplied by 10 (or which is all one, remove the Prick one place toward the Right-hand) the Product is 845.3, the whole Content of the Tun.

Note, Here the Diameters being taken in the middle of every 10 Inches, there will be no uſe of Multiplication or Diviſion in Inching the Tun, as there will be when the Diameters are taken in the middle of every Foot or half Foot; for here the Area's of the ſeveral Diameters do not only ſhew the mean Inch in the middle of every 10 Inches but alſo the whole Content of thoſe 10 Inches, if you remove the Prick but one Place more towards the Right-hand. Thus the mean Inch of the laſt 10 Inches is 31.00, but if the Prick be removed as aboveſaid, it is 310, which is the Content of a Tun at 10 Inches deep; I therefore reduce the ſeveral Contents in the third Column into Barrels, Firkins, and Gallons, Beer-meaſure, and ſet them down in the fourth Column; theſe added together, are 23 Barrels, 1 Firkin, 8 Gallons, and 3 tenths of a Gallon, which agrees with the Sum of the Numbers in the third Column.

Parts of the Tun's Depth	*Mean Diameters in the middle of every 10 Inches of the Tun's Depth.*	*Mean Area's or Contents of every 10 Inches of [illegible] Tun's Depth*	*Content of every 10 Inches Beer-Meaſure.*		
			B	F.	G
10	95.5	25.40	7	0	2.0
10	100.5	28.13	7	3	2.3
10	105.5	31.00	8	2	.4.
30		84.53	23	1	8 3

By theſe Numbers thus found it will be eaſy to make a Table, which ſhall ſhew the Quantity of Liquor contain'd in the Tun at any Depth, of which obſerve the following Directions.

1. Set down the whole Content of the Tun in Barrels, Firkins, Gallons and Parts, in their reſpective Columns, and at the Top of the firſt Column againſt the Content ſet 0, and under it the Tun's Depth in Inches, 1, 2, 3, 4, 5, *&c.* this done take the firſt mean Inch, *viz.* 25,4, and reduce it into Barrels, Firkins, and Gallons, Beer-Meaſure, it makes 0 : 2 : 7 : 5; ſet this upon a Scroll of Paper, and ſubſtract it continually from the whole Content till you come at the tenth Inch of the Tun's Depth; which done if you miſtake not, the Remainder there will be 16 : 1 : 6 : 3.

2. Take the ſecond mean Inch, *viz.* 28, 13, this being reduced as the former is 0 : 3 : 1 ; 1; ſubtract this continually from the laſt Remainder, till you come at 20 Inches deep, and there your Remainder will be 8, 2, 4.

Take the third and laſt mean Inch, and reduce and ſubtract it as before, and if your Work be right, there will be no Remainder at the laſt Inch or Bottom of the Tun; now the Table being made, you may caſt away the Fractions as uſeleſs. The uſe of this Table is obvious to the meaneſt Capacity, without any further Explanation:

In	P.	F.	G.
0	23	1	8.30
1	22	3	0.9
2	22	0	2.5
3	21	1	4.1
4	20	2	5.7
5	19	3	7.3
6	19	0	8.9
7	18	2	1.5
8	17	3	3.1
9	17	0	4.7
10	16	1	6.3
11	15	2	5.17
12	14	3	4.04
13	14	0	2 91
14	13	1	1 78
15	12	2	0.65
16	11	2	8.52
17	10	3	7.39
18	10	0	5.26
19	9	1	5.13
20	8	2	4.00
21	7	3	0
22	6	3	5
23	6	0	1
24	5	0	6
25	4	1	2
26	3	1	1
27	2	2	6
28	1	2	3
29	0	3	4
30	0	0	0

For if you come to this Tun, and find 2 Inches of the Depth dry, the Quantity of Liquor then in the Tun is 22 B 0 F. 2 G. If 7 Inches be dry, there remain in the Tun but 18, 2. 1, *&c.* Thus for Round Tuns.

Now, ſuppoſe the Tun laſt mention'd had been Square, and the Sides at Bottom and Top equal to the Diameters of the laſt, and the Depth the ſame; the Diameters or Sides in the middle of every 10 Inches of the Tun's Depth will be the ſame as in the laſt; by which the ſeveral mean Inches, and alſo the whole Contents of every 10 Inches are found to be as in this Table.

Parts Tun's Depth.	*Side of the Tun i the middle of every ten Inches.*	*Mean Area's or Contents of every ten Inches.*	*Content of every 10 Inche Beer-Meaſure.* B.	F.	G
10	25 5	32.34	8	3	8 4
10	100.5	[illegible]81	9	3	7.1
10	10; 5	39.46	10	3	7.6
30			29	3	5.11

For ſet 282 upon B, to 9;.5 the firſt Diameter upon A, then againſt 9 .5 upon B, is 3 .34 the firſt mean Inch, and after the ſame Manner find the ſecond and third, which being reduced as in the laſt Example, I find the whole Content to be 1075.1 Ale Gallons (or which is the ſame thing) 29 B. 3 F. 5 1 G. This being done, you may proceed to inch this Tun by the ſame Directions which were given in the laſt Example.

PROB. III.

To Inch a Tun whoſe Baſes are unlike, viz. *the lower Circular, and the upper Elliptical, ſuppoſe the Diameter of the Circular Baſe be* 100 *Inches, the Axis (or longeſt Diameter) of the Elliptical Baſe* 100, *and the ſhorteſt* 90, *and the Tun's Depth* 20 *Inches.*

TO Inch this Tun. I take the Croſs Diameters to the middle of every 4 Inches of the Tun's Depth: Thus at 2 Inches from the Top I find the longeſt Diameter 100, and the ſhorteſt 91, in the Middle of the next four Inches (which is 6 Inches from the Top) the longeſt Diameter is 100, and the ſhorteſt 93; and thus I find the reſt, which are as in this Table is expreſs'd: Now having theſe Diameters, the Area or mean Inch in the middle of every 4 Inches of the Tun's Depth may be thus found.

Set 359.05 upon B, to the longeſt Diameter, (*viz.* 100) upon A, then againſt the ſhorter Diameter (*viz.* 91) upon B, is 25.34, the firſt mean Inch, or Area in Ale-Gallons; and after the ſame Manner the ſecond and third will be found, as in the following Table.

Depth.

Depth.	*Axis.*	*Shorter Dia-*	*Mean Inch in Ale-Gallons.*	*Content of every 4 Inches of the Tun's Depth.*		
4	100	91	25 34	2	3	2 36
4	100	93	25.88	2	3	2 52
4	100	95	26.47	2	3	6.88
4	100	97	27.02	3	0	0.08
4	100	99	27.57	3	0	2.23
20			132.28	14 .	2 .	4.12

Now if the Mean Inches be ſeverally multiplied by 4, and theſe Products reduced into Barrels, Firkins, and Gallons, Beer-meaſure, they will give the Numbers in the laſt Column; which added together are 14 Barrels, 2 Firkins, 4 Gallons, and 12 Parts of a Gallon.

Set this down to the firſt Number of your Table, and then reduce the firſt mean Inch into Barrels, Firkins, and Gallons, and ſubſtract it 4 times from the whole Content and Remainders; do the like with the other mean Inches, and ſo compleat the Table as in the laſt Example.

PROB. IV.

To find the Drip or Fall of a Tun, and to make Allowance for the ſame in Inching the Tun.

Example.

ADmit the Tun laſt mentioned were ſo placed, that when the Bottom is but juſt covered on one ſide, the Liquor is 4 Inches deep on the ſide oppoſite ; How much muſt be allow'd for the Fall of this Tun ?

By the Operation aforegoing the Area of the mean Diameter in the middle of the 4 laſt Inches (that is, at 2 Inches from the Bottom) was found to be 27.57, this multiplied by 4, is 110.28 Ale-Gallons, the half of this is 55.14, and ſo much may be allow'd for the Fall of the Tun ; reduce theſe 55.14 Gallons into Barrels, Firkins, and Gallons Beer-meaſure gives 1 : 2 : 1. 14 : and this taken from 14 : 2 : 7. 12, the Content of the Tun when it ſtands level, there remains 13 Barrels, 0 Firkins, 5 Gallons, and 98 parts : And ſo much will the Tun contain, when the Fall is 4 Inches ; therefore ſet this at the Top of the Table, then reduce the firſt mean Inch, *viz.* 25,34 into Barrels, Firkins, and Gallons, and ſubtract it four times from the whole Content 13 : 0 : 5 : 9.8 the Remainder will be 10 : 1 : 3 : 2 ; Do the like with ſecond, third, and fourth mean Inches, and you will find the Remainder at 16 Inches dry, to be 1 : 1 14 : And ſo much it takes to cover the Bottom, as was before obſerved.

0	13	0	5.98
1	12	1	7.64
2	11	3	0.30
3	11	0	1.96
4	10	1	3.62
5	9	2	4.74
6	8	3	5.86
7	8	0	6.98
8	7	1	8.10
9	6	2	8.63
10	6	0	0.16
11	5	1	0.69
12	4	2	1.22
13	3	3	1.20
14	3	0	1.18
15	2	1	1.16
16	1	2	1.14

As for ſuch Tuns as are ſuppoſed to be the Fruſtrums of *Spheroids*, *Parabolic Conoids*, *Hyperabolic Conoids*, or *Parabolic Spindles*, I ſhall give Rules for Gauging and Inching of theſe in the Appendix, where I ſhall alſo endeavour to explain the Nature of theſe Figures, and ſhow whence they are derived.

PROB.

PROB. V. Fig. 20. Plate III.

To Gauge a Copper, *and make Allowance for the Crown: let* a b c d e, *repreſent a* Copper, *whoſe Content is required.*

1. TO find the ſeveral Diameters and Depth of the Copper, together with the Altitude of the Crown,

Take a ſmall Cord, or Thread, make one End faſt at *a*, and extend the other to the oppoſite Side of the Copper at *e*, where make it faſt, or cauſe ſome Perſon to hold it very ſtraight; then ſet one End of the Inſtrument in the Bottom of the Copper at *b*, and move it to and fro, till you find the neareſt Diſtance to the Thread (as at *o*,) this Diſtance (*b o*) is the Depth of the Copper, ſuppoſe it to be 44 Inches.

In like Manner, ſet the End of the Rule upon the Top of the Crown at *c*, and take the neareſt Diſtance to the Thread (as *c v*) ſuppoſe it be 36 Inches; this ſubtracted from (*b o*) 44, the Remainder 8, is the Altitude of the Crown.

To find (*b d*) the Diameter of the Bottom of the Crown.

Meaſure (*a e*) the Diameter at the Top, admit it be 115 Inches; then hold a Thread ſo as the Plummet at the End thereof may hang juſt over *b*; by this Means you may find the Diſtance *a o*, and on the other Side *e s*, ſuppoſe each be 14. 5; add theſe together, and ſubtract their Sum (*viz.* 29) from *a e* 115, the Remainder 86, is equal to *b d*, the Diameter at the Bottom of the Crown: The Diameter, which touches the Top of the Crown, may be found by the Inſtrument, ſuppoſe it be 91.4 Inches.

Now, to find the Content of the Copper from the Crown upwards (that is, the Part *a b c i e*) the Depth (*v c*) being 36 Inches, you may take a Diameter in the Middle of every 6 Inches betwixt *v* and *c*, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective *Area*'s in Ale-Gallons, found by the Gage-Point as before; the fourth Column ſhews the Content of every 6 Inches in Barrels, Firkins, and Gallons, Beer meaſure: Theſe added together, are 29 Barrels, 3 Firkins, 2.5 Gallons.

And ſo much will the Copper contain after the Crown is cover'd.

Now, if the Crown be taken for the Fruſtum of a Sphere, the Content (by the third Problem of *Section* V.) will be found to be 83.3 Gallons.

But the Content may be more readily found, very near the Truth. Thus,

The Diameter *b d* was found to be 86, the Area to this Diameter is 20.6, this multiplied, by half the Crown's Altitude, *viz.* 4. gives 82.4, the Content of the Crown, this ſubtracted from the Content of the Part or Portion *b c i d n b*, the Remainder is the Quantity of Liquor that will cover the Crown; the Content of the Part *b c i n d b* may be thus found, *b i* is 91.4, and *b d* is 86, theſe added together are 177.4, the half 88.7, may be taken for a mean Diameter, the Area to this Diameter is 21.9; this multiplied by 8, the Altitude, gives 175.2, the Content.

Now ſubtract 82.4 (the Content of the Crown) from 175.2, the Remainder is 92.8, or 2 Bar. 2 Fir. 3 Gall. and .28 of a Gall. this added to the Content firſt found, *viz.* 29 Bar. 3 F. 2.5 Gall. gives 32 B.

1 F. 5.3 Gall. the Content of the whole Copper, as in the following Table.

Parts of the Depth.	*Diameter.*	*Mean Inch.*	*Content of every* 6 *Inches.*		
			B.	F.	G.
6	113.1	35.62	5	3	6.7
6	109.2	33.21	5	2	1 2
6	105.28	30 86	5	0	5.1
6	101.36	28.61	4	3	0.6
6	97.33	26.37	4	1	5.2
6	93.4	24.29	4	0	1.7
36	Sum ——— ——— ———		29	3	2 5
To cover the Crown ———			2	2	2.8
Content of the Copper ———			32	1	5.3

The Content being thus found, you may proceed to inch the Copper by the ſame Directions which were given for Inching of Tuns in the laſt Problem.

PROB. VI.

To Guage a Still.

Let Fig. 22. Plate II. *repreſent a* Still *whoſe Content is required:*

TAKE with a Line or Piece of Packthread the Diameter of the Collar W w, and from X the Centre of it, take the Depth of the Still, or Line X x, with your Rule. Keep your Rule fixed in the ſame Poſition, and extend the Line from A L, obſerving where it interſects the Rule, that is, find how many Inches is contained between X and the Interſection of the Line with the Rule at B. Fix the End of the

Line in the Point *x*, and with the other End of the Line, having a Piece of Chalk fixed to it, deſcribe a Circle; quarter it, and extend the Diameters to the Sides of the Still. Faſten one End of the Line in the Point *y*, and let your Aſſiſtant extend the other to the Point A. Faſten a Piece of Chalk in the End of a Stick, and guiding the Stick by the Side of the Line up the Side of the Still from *y* to A. Do the ſame from the Points where the Diameters of the Circle deſcribed on the Bottom interſect the Sides of the Still. In the Middle of every 10 Inches, from A and L towards K and V make a Mark with your Chalk, and meaſure the ſeveral Diameters from Mark to Mark, as A L, B, M, *&c.* together with the croſs Diameters *a l*, *b m*, *&c.* becauſe few Stills are accurately round. Let the ſeveral Diameters be as follows:

$$\left.\begin{array}{r} W\,w\text{, the Collar} = 40 \\ A\,L = 130 \end{array}\right\}\ \text{the Segment of a Globe.}$$

B M = 137 *b m* = 136, mean Diameter = 136,5
C N = 147,2. *c n* = 146,1 mean Diameter = 146,65
D O = 153.3. *do* = 152,6 mean Diameter = 152,95
E P = 154,2. *ep* = 153,7 mean Diameter = 153.95
F Q = 153,1 *fq* = 152,4 mean Diameter = 152,75
G R = 151,2. *gr* = 150,7 mean Diameter = 150,95
H S = 148,5. *hs* = 147,8 mean Diameter = 148,15
I T = 140,6. *it* = 139,7 mean Diameter = 140,15

The whole Depth X *x* = 106,5 Inches, and X B = 25.4.

Areas of the mean Diam. — *Contents in Wine Gall.*

6 ,37 × 10 - - - = 633,7
73,15 × 10 - - - = 731,5
79,57 × 10 - - - = 795,7
80,65 × 10 - - - = 806,5
79,37 × 10 - - - = 793,7
77,47 × 10 - - - = 774,7
74,75 × 10 - - - = 747,6
66,81 × 11, 1 - - = 741,59
Content of the Segment of a Sphere = 1006,54
Content of the Crown or Bottom = 362,2

Content of the Still in Wine Gall. 7393,73

The Content of the Segment of a Sphere A W X *w* L may be found by Prob. III. Page 102, or thus; to the Area of the Collar W *w*, add twice the Area of the Diameter A L, and multiply that Sum by ⅓ of the Depth Y B = 8,36, the Product will be the Content required. Thus,

The Area of the Diameter A L equal to 57,48
Which being multiplied by —— 2

114,96
The Area of the Collar W *w* equal to 5,44

120,40
Which being multiplied by —— 8,36

72240
36120
96320

Gives the Content of the Segment equal to 1006,5440

In the preceding Calculation, the mean Diameters are found by adding the two croſs Diameters together and taking half that Sum. And the Area of the mean Diameters by ſquaring them (that is multiplying them into themſelves) and dividing that Product by 294, the circular Diviſor.

The beſt Method, and indeed what is generally made uſe of, to find the Content of the Crown or Bottom, is to meaſure into it, by a regular Veſſel, a Quantity of Water that will juſt cover it. But the Content of it may alſo be found by the foregoing Problem, to which we refer the Reader.

The Content of the Still being thus found, you may Inch it by the Directions given in Prob. IV. preceding.

PROB. VII.

The Diameter of any Round, or Side of any Square Maſh-Tun, being given, to find how many Gallons of Malt it will contain at one Inch deep.

THE Corn-Gallon (which is the Meaſure we here account by) is ſuppoſed to contain 268.8 cubic Inches.

But Mr. *Dary*, in his Complete Gauger, ſaith, that an indifferent Sort of Malt, and of an indifferent Grinding, when three Worts ſhall have paſſed through it, will be more compact by about a ſixth Part; he therefore concludes, that 227 may be a proper Diviſor for ſquare Maſh-Tuns, and 388 for round ones.

1. *For Round Maſh-Tuns.*

The Diameter of that Circle, whoſe Area is 227, is 17, and this may be called the *Gage-Point* for Round Maſh-Tuns; and to find the Content of any ſuch Maſh-Tun,

Set 17 upon D, to 1 upon C; then againſt any Diameter upon D, you have the Contents in Gallons at one Inch deep. Thus, if the Diameter be 113, the Area or Content at one Inch deep would be 44.3, that is, 5 Buſhels, and 4 3 Gallons.

2. *For Square Maſh-Tuns.*

If a Maſh-Tun be Square or Oblong, the Content at one Inch deep may be thus found: Say,

As 227 is to the Length;
So is the Breadth to the Content.

Example. Suppoſe the Length 112 Inches, and the Breadth 80, how many Gallons will this Tun contain at one Inch deep? Anſwer, 39.4: For,

Set 227 upon A, to 80 upon B; then againſt 112 upon A, is 39.4, the Content upon B.

PROB.

PROB. VIII.

To Gauge a Diſtiller's Back.

LET Fig. 21. Plate II. repreſent a Diſtiller's Back, of an Elliptical Form, its Baſes unequal but alike ſituated.

In order to find the Content of this Back, the ſeveral Diameters muſt be firſt found in the following Manner: Firſt, find the tranſverſe or longeſt Diameter *m n* with a Line, marking where it interſects the Sides, as at *m*, and *n*, and chalking your Line ſtrike on the Bottom the Line *m n*, and from *m* and *n* draw the Lines *m* A, *n a*, perpendicular to *m n*; divide the tranſverſe Diameter, or Line *m n*, into two equal Parts, as at *z*, which will be the Center of the Baſe, or Place where the conjugate and tranſverſe Diameters interſect each other; thro' *z* draw the conjugate Diameter at Right-Angles to the Tranſverſe, which may be done by Help of a T Square, or a common Quadrant, by applying one Limb to the Line *z n*, and drawing a Line by the other; make a Mark where the Ends of the conjugate Diameters interſect the Sides of the Back, and draw Lines up the Sides of it perpendicular to the conjugate Diameter. In the Place where the Officer propoſeth to take his Gauge, which ſuppoſe at *o*, make a Notch, and take the Depth *o z*, equal to 30 Inches; divide the Lines A *m*, *a n*, into 6 equal Parts, each of which will be equal to 5 Inches, (becauſe the whole Depth is 30 Inches) and in the Middle between thoſe Marks, meaſure, by the Help of your Line, the tranſverſe and conjugate Diameters, which ſuppoſe as follows:

Tranſverſe

Tranſverſe Diameters.	*Conjugate Diameters.*
B *b* = 50	P *p* = 30
C *c* = 53	R *r* = 31
D *d* = 57	T *t* = 33
E *e* = 60	V *v* = 35
F *f* = 63	W *w* = 36
G *g* = 67	X *x* = 38

Having taken the Diameters as above, before we ſhew the Method of finding the Areas of thoſe Diameters, we ſhall ſhew how to find the *Drip* of the Back. The beſt Way to perform this is to meaſure into it, by a regular Veſſel, ſuch a Quantity of Water as will juſt cover the Bottom of the Back, which ſuppoſe 14 Gallons; then dip it with your Rule in ſeveral Places to find how much it wets, which ſuppoſe 2 Inches.

Having found the Drip, proceed to find the Areas of the ſeveral Diameters thus; multiply the Tranſverſe Diameter by the Conjugate, and divide that Product by 294, (the Circular Diviſor) the Quotient will be the Area required. And this Area multiplied by 5, the Product will be the Content of the reſpective Part of the Back to which the ſaid Diameters belong. See the Work.

Tranſverſe Diameter.	*Conjugate Diameter.*	*Area in W. Gal.*	*In.*	*Cont. of the Depths in W. Gal.*
50 - - - -	30 - - -	= 5,1	× 5	= 25,5
53 - - - -	31 - - -	= 5,59	× 5	= 27,95
57 - - - -	33 - - -	= 6,39	× 5	= 31,95
60 - - - -	35 - - -	= 7,16	× 5	= 35,8
63 - - - -	36 - - -	= 7,72	× 5	= 38,6
67 - - - -	38 - - -	= 8,66	× 3	= 25,98
			Drip	= 14

Content of the Back —— 199,78

Note, The Laſt mean Area is multiplied by 3 Inches, becauſe the Depth of the Drip is two Inches, which being ſubtracted from 5 Inches, leaves 3, the Depth by which the laſt Area muſt be multiplied.

From the foregoing Calculation, it appears that the Content of the Back is 199,78 Gallons.

But the Content of Backs of this Form may alſo be found by the help of a Mean Proportional, which is found by multiplying the two Extremes together, and extracting the Square Root out of the Product: Thus the two extreme tranſverſe Diameters (or thoſe at the Top and Bottom) are 50 and 67, which being multiplied together, and the Square Root of that Product extracted, gives 57,87 for a mean tranſverſe Diameter: Alſo the extreme conjugate Diameters are 30 and 38, whence the mean conjugate Diameter will be 33,76; the Area of this Ellipſis is 6,65 Wine Gallons, which multiplied by 28 Inches, the whole Depth, and the Drip added to that Product, gives 199,92 Gallons, the Content of the Back, nearly the ſame as before, and indeed all Backs whoſe Sides are ſtraight from Top to Bottom, are better gauged by this Method, if nothing more be required than to find their Contents, as the Anſwer is nearer the Truth, and at the ſame time may be performed with half the Trouble; but if the Sides of the Back are not directly ſtraight, the firſt Method is to be preferred, it being equally true when the Sides form a Curve as when they are ſtraight.

To Tabulate the Back.

From the whole Content of the Back ſubtract the Area of the mean Diameters as often as you multiplied them, (*viz.* 5 times each, except the laſt) and the Remainders will be the Contents of the Back at any dry Inch from Top to Bottom, which you will find as follows:

In.	*Gall.*	*In.*	*Gall.*	*In.*	*Gall.*	*In.*	*Gall.*	*In.*	*Gall.*	*In.*	*Gall.*
Full	199,78	6	168,69	11	139,94	16	107,22	21	70,86	26	31,32
1	194,68	7	163,10	12	133,55	17	100,06	22	63,14	27	22,66
2	189,58	8	157,51	13	127,16	18	92,90	23	55,42	28	14,
3	184,48	9	151,92	14	120,77	19	85,74	24	47,70	30	00,
4	179,38	10	146,43	15	114,38	20	78,58	25	39,98		
5	174,28										
Area 5,1		Area 5,59		Area 6,39		Area 7,16		Area 7,72		Area 8,66	

SECT.

SECT. IX.

Of Caſk-Gauging.

BEFORE we can find the Content of a Caſk, we muſt know the Dimenſions of it, *viz.*

The { Bung / Head / Length } Diameter } in Inches.

And when theſe are known, the Form or Shape of the Caſk muſt be conſidered; for although the Diameters (aboveſaid) and the Length of one Caſk may be equal to thoſe of another, yet one of theſe Caſks may contain ſeveral Gallons more than the other, and therefore the Content of all Caſks cannot be found by one and the ſame Rule.

Now, moſt Writers, in treating of this Subject, have taken it for granted, that every common Caſk, (as Butt, Pipe, Hogſhead, Barrel, and ſuch like) is in the Form of one of theſe Solids, *viz.*

1. The middle Fruſtum of a Spheroid.
2. The middle Fruſtum of a Parabolical Spindle.
3. The middle Fruſtum of two Parabolical Conoids abutting upon one common Baſe; or,
4. The middle Fruſtum of two Cones, abutting upon one common Baſe.

Accordingly they have laid down Rules for finding the Contents of theſe Solids, and by thoſe Rules they ſuppoſe the Content of any Cask may be found.

As for Inſtance.

If the Dimenſions of a Cask be

Bung	Diameter	32	Inches.
Head	Diameter	24	Inches.
Length		40	Inches.

By the Rule given by Mr. *Dary*, and others, the Content of this Cask, if taken for

				Ale-Gall.
The middle Fruſtum of	a	Sphroid	is	97.455
	a	Parabolic. Spindle	is	96.505
	2	Parabolic. Conoids	is	89.136
	2	Cones	is	87.934

The Rules are true, as to the firſt and laſt of theſe Solids, *viz.*

The middle Fruſtum of | a Spheroid,
The middle Fruſtum of | Two Cones.

And the may be repreſented by Figure 21. Plate III.

A Cask, which is the middle Fruſtum of a Spheroid, is ſhewn by the outermoſt Line *d o a d f c o b o c e d*, the two ſtraight Lines *d e c*, and *d f e*, are the Heads of the Csak, and the curved Lines *d o a o d*, and *c o o e*, the Staves.

If a Caſk be in the Form of the middle Fruſtum of two Cones, it may be repreſented by the ſtraight Lines *d u a u d*, and *c u b u c*, the Heads *d e c*, and *d f c*, the Bung-Diameter *a b*, and the Length *c f*, are all the ſame in both theſe Caſks; and yet 'tis evident by the Figure itſelf, that one will contain more than the other; and by the Example above cited, the Difference is 9.521 Gallons; for the Spheroid contains 97.455 Gallons, and the Fruſtum of the two Cones but 87.934.

There are very few (if any) Caſks, whoſe Diameters and Length are equal to thoſe abovenamed, that will contain more than 97.455 Gallons (the Contents of the Spheroid) nor leſs than 87.934 (the Content of the Fruſtum of two Cones;) ſo that all the Varieties that can happen, will fall betwixt theſe two; and the Staves of any Caſk differing from theſe, will lie between the Line *d o a*, and *d u a*, in Figure 21.

Now, for the other two Solids, *viz.* the middle Fruſtum of a Parabolic Spindle, and the middle Fruſtum of two Parabolic Conoids; according to the Authors above-cited, the Content of the firſt is 96.505 Gallons, and the ſecond 89.136.

Whether the Rules they have given for finding theſe Contents be true or falſe, I ſhall examine in the Appendix.

But ſuppoſing they are true, yet they do not anſwer the various Caſes that may happen in common Practice; for in the Example above, the Content of a Caſk, taken as the middle Fruſtum of a Parabolic Spindle, is but .95 of a Gallon leſs than the Content of the Fruſtum of a Spheroid, and the Content of the Fruſtum of two Parabolic Conoids is but 1.931 more than the Fruſtum of two Cones.

Now, I preſume, no Man will pretend to diſtinguiſh which Notion a Caſk muſt be taken in, whether as the Fruſtum of a Spheroid, or as the Fruſtum of a Parabolic Spindle, when the Contents differ not ſo much as a Gallon.

Moreover, here is above 7 Gallons Difference betwixt the Contents of the Fruſtum of a Parabolic Spindle, and the Fruſtum of two Parabolic Conoids: So that I may modeſtly affirm, there are many Caſks whoſe Content cannot be found by any of theſe Rules: for it muſt be granted, that a Caſk, whoſe Diameters and Length are equal to thoſe in this Example, may contain ſome certain Quantity betwixt 96 Gallons, and 89.

As ſuppoſe 93; I ſay, if this Caſk were to be Gauged by any of the Rules above-cited, it will give the Content at leaſt 3 Gallons too much, or 4 Gallons too little.

This premiſed, I come now to lay down ſome ſhort Rules, by which the Content of any Cask may be found very near the Truth.

PROB. I. Fig. 21. Plate III.

There is a Cask ſuppoſed to be the middle Fruſtum of a Spheroid, intercepted between two Plane, Parallel, cutting the Axis at Right Angles: let a b, *the Diameter at the Bung, be* 32 *Inches,* d c, *the Diameter at the Head,* 24, *and the Length* e f 40 *Inches: How many Ale or Wine-Gallons will this Cask contain?*

RULE.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add half the Difference of the ſaid Squares; the Sum of theſe multiplied by the Length, and this Product divided

$$\text{By}\left\{\begin{matrix}1077\\882\end{matrix}\right\}\begin{matrix}\text{Quotes the}\\\text{Content in}\end{matrix}\left\{\begin{matrix}\text{Ale}\\\text{Wine}\end{matrix}\right\}\text{Gallons.}$$

EXAMPLE.

Square of *a b* 32 is	1024
Square of *d c* 24 is	576
Sum is	1600
Half Sum	800
Half the Difference of the Squares	224
Sum is	2624

Which multiplied by 40 (the Length of the Caſk) gives 104960; this divided by 1077, quotes 97.455, the Content in Ale-Gallons.

Or divided by 882, the Quotient will be 119.002, the Content in Wine-Gallons.

Note, The common Rule for Gauging the Fruſtum of a Spheroid (by taking twice the Square of the Bung-Diameter, and the Square of the Head Diameter, *&c.*) is ſomewhat a ſhorter Way than this: But this Rule being more agreeable to the Work in the following Problems, I have thought fit to inſert it here. The Reader may uſe which he pleaſes.

PROB. II.

Suppoſe a Caſk (of the ſame Diameters, and Length with the former) ſhall have its Staves betwixt the Bung and Head leſs curv'd, ſo as to leave out about a third Part of the Space between the Lines d o a *(which repreſent the Spheroid) and* d u a, *as in* Fig. 22. Plate III. *How many Gallons will this Caſk contain?*

RULE.

TO the Sum and half Sum of the Squares of the Bung and Head-Diameters, add three tenth Parts of the Difference of the ſaid Squares; the Sum of theſe multiplied by the Length, and the laſt Product divided by 1077, or 882, gives the Content in Ale or Wine-Gallons reſpectively.

Example. *The Dimenſions as above.*

Sum of the Square is ——— ——— 1600
Half Sum is——— ——— 800
The Difference of the Squares multiplied by .3, is 134.4

Sum of theſe is ——— ——— 2534.4

This multiplied by 40, gives 101376; and this divided by 1077, quotes 94.128, the Content in Ale-Gallons, or by 882, the Quotient will be 114.938, the Content in Wine-Gallons.

PROB. III. Fig. 19. Plate III.

There is a Cask, whoſe Staves between the Bung and Head have but little Curvature, lying nearer to the ſtraight Line a u d, *than to the Curved Line* a o d. *What is the Content of this Cask in Ale or Wine Gallons?*

RULE.

TO the Sum and half Sum of the Squares of the Bung and Head-Diameters, add one tenth of the Difference of the Squares; the Sum of theſe multiplied by the Length, and this Product divided as above directed, gives the Content in Ale or Wine-Gallons reſpectively.

Example. *The Dimenſions as above.*

Sum of the Squares is ———————— 1600
Half Sum is ———————————— 800
One tenth of the Difference is ———— 44.8

Sum of theſe is ——————————— 244.8

Which multiplied by 40, gives 97792; this divided by 1077, the Quotient is 90.800. the Content in Ale-Gallons; or by 882, gives 110.875, the Content in Wine-Gallons.

PROB. IV. Fig. 21. Plate III.

To find the Content of a Cask in form of the middle Fruſtum of two Cones abutting upon one common Baſe (if any ſuch can be) it may be repreſented by the ſtrait Lines d u a, a u d.

RULE.

FRom the Sum and half Sum of the Squares of the Bung and Head Diameters, ſubtract half the Square of the Difference of the Diameters themſelves; Multiply the Remainder by the Length, and divide this Product as in the laſt, the Quotient will be the Content in Ale or Wine Gallons.

Example. *Dimenſions as above.*

Sum of the Squares is ——————— 1600
Half Sum is ——————— 800

Sum of theſe is ——————— 2400
Difference of the Diameter is 8, this ſquared is 64, the half is ——————— 32

The Remainder is ——————— 2368

This multiplied by 40, gives 94720. and this divided by 1077 quotes 87.948, the Content in Ale-Gallons; or 882 the Quotient will be 107.392 the Content in Wine-Gallons.

Thus it appears by the four Problems aforegoing, if the Dimenſions of a Cask

Be Bung Diameter ——————— 32 Inches.
Be Head Diameter ——————— 24 Inches.
Be Length ——————— 40 Inches.

The Content in each Variety is as Followeth;

By Problem	Ale-Gall.	Diff.
I.	97.455	
II.	94.128	3.327
III.	90.800	3.328
IV.	87.948	2.852

And here it may be obſerved, that the Differences of the Contents thus found are very near equal: So that by theſe Rules there is not ſo much Room for Error, as by the Rules which ſeveral Authors have preſcribed.

I have inſiſted the longer on this Buſineſs, that I might prevent Miſtakes, for we cannot be too exact, where a ſmall Error muſt needs be prejudicial either to the King, or to the Merchant, eſpecially when the Duty upon Brandy is great.

PROB. V.

The Dimenſion of any Cask being given to find the Content by the Inſtrument.

TO do this, we muſt firſt find a mean Diameter, that is ſuch a Diameter as ſhall reduce the Cask to a Cylinder; for which Purpoſe I have calculated theſe three Tables, for the three firſt Varieties of Casks abovementioned.

A TABLE for Equating the Diameters of the middle Fruſtum of a Spheroid.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.67	0.73	0.80	0.82	0.94	1.01	1.08	1.15	1.22	1.29
2	1,36	1.43	1.50	1.57	1.64	1.71	1.78	1.85	1.92	1 99
3	2.06	2.13	2.20	2.27	2.34	2.41	2.48	2.55	2.62	2.69
4	2.76	2.83	2.90	2.97	3.04	3.11	3.18	3.25	3.32	3.39
5	3.46	3.54	3.61	3.68	3.75	3 82	3.89	3 96	4.03	4.10
6	4.17	4 24	4.31	4.39	4.46	4.53	4.60	4.67	4.74	4 81
7	4 88	4.95	5.03	5.10	5.17	5.25	5.32	5.39	5.46	5.53
8	5.60	5.68	5.75	5.82	5.90	5 97	6.04	6.11	6.18	6.25
9	6.32	6 40	6.47	6.55	6.62	6.70	6.77	6.84	6.91	6.98
10	7.05	7.13	7.20	7.28	7.35	7.47	7.49	7.56	7.64	7.71

A TABLE for the Second Variety of Casks.

	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.62	0.68	0.74	0.81	0.87	0 93	0.09	0.06	1.12	1.18
2	1.24	1.30	1.36	1.43	1 49	1.55	1.62	1.68	2.74	1.80
3	1.86	1.93	1.99	2.05	2 12	2.18	2.24	2.30	2.37	2.43
4	2 49	2.55	2.62	2.68	2.75	2.81	2.88	2.94	3 01	3 07
5	3.13	3 20	3.26	3.33	3.39	3 46	3 52	3.59	3.65	3.72
6	3.78	3.85	3.91	3.98	4 04	4 11	4.18	4.24	4.31	4 37
7	4.44	4.50	4.57	4.63	4.70	4.76	4 83	4.90	4 96	5 03
8	5.10	5.16	5.23	5.30	5.36	5.43	5.49	5.56	5.63	5.70
9	5.77	5.84	5.91	5.97	6.03	6.10	6.17	6.24	6.30	6 31
10	6.44	6.51	6.57	6.64	6.71	6.77	6 83	6.90	6 96	5.02

A Table for the Third Variety of Casks.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.55	0.60	0.65	0.70	0.76	0.81	0.87	0.93	0.99	1.05
2	1.11	1.17	1.22	1.28	1.33	1.39	1.44	1.50	1.55	1.61
3	1.67	1.73	1.78	1.84	1.90	1.95	2.01	2.06	2.12	2.18
4	2.24	2.30	2.35	2.41	2.46	2.52	2.57	2.63	2.69	2.75
5	2.81	2.87	2.93	2.98	3.04	3.20	3.16	3.21	3.27	3.33
6	3.39	3.45	3.51	3.56	3.62	3.68	3.73	3.80	3.85	3.91
7	3.97	4.02	4.08	4.14	4.20	4.26	4.32	4.38	4.44	4.50
8	4.56	4.62	4.68	4.74	4.80	4.86	4.92	4.98	5.04	5.10
9	5.16	5.22	5.28	5.35	5.41	5.47	5.53	5.59	5.65	5.71
10	5.77	5.83	5.89	5.96	6.02	6.08	6.14	6.20	6.26	6.32

Now by theſe Tables the mean Diameter of any Cask is thus found.

Subtract the Head-Diameter from the Bung-Diameter, then ſeek the Difference in the Table, taking the whole Inches in the firſt Column, and the Tenths (if any be) at the Head ; in the Angle of meeting you have a Number, which being added to the Head-Diameter, gives the mean Diameter ſought.

Example. Let the Dimenſions be as in the firſt Problem of this Section, the Difference of the Diameter is 8. In the Table for a *Spheroid*, againſt 8 on the Side, and under 0, I find 5.6; this added to the Head-Diameter (*viz.* 24.) the Sum is 29.6, the Diameter ſought.

This being obtained, the Content of any Caſk may be found by the Inſtrument at one Operation, for which the Proportion is,

As the Gage-point is to the Caſk's Length;
So is the mean Diameter to the Content.

Example. The Length 40, and the mean Diameter 29.6 Inches.

Set the Gage-point for Ale-Gallons to 40 upon C, then againſt 29.6 upon D, is 97.455, the Content in Ale-Gallons, which is the ſame as was found by the firſt Problem of this Section.

Note, The three Tables above are expreſſed in three Scales, and placed upon one Side of the Rule againſt the Line of Inches, by which the mean Diameter of any Caſks may be readily found. See Plate I.

To find the Ullage of a Caſk.

PROB. VI.

Let Figure 21. *Plate* III. *repreſent a Caſk poſited with its Axis* e f, *Parallel to the Horizon, in part Empty, the Surface of the Liquor cutting the Bung Diameter in S, ſuppoſe the whole Content* 87.455 *Gallons the Bung-Diameter* a b, 32, *Inches, the wet Part* s b, 24, *the dry Part* s a, 8; *How many Gallons are contained in the Cask; And how many will it require to fill it full?*

FOR reſolving this, and ſuch like Queſtions, there is a Line of Segments on the Rule, See Plate I. with a Line of Numbers to ſlide againſt it, and upon theſe Lines the Proportion is,

1. *As the Bung-Diameter upon the Number, is to* 100 *upon the Segments;*
So is the wet or dry Part in the Numbers, to a fourth Number on the Segments.

2. *As* 100 *upon* C, *is to the Cask's whole Content upon* A;
So is the fourth Number laſt found to the Anſwer.

Example. 1. Set 32 the Bung-Diameter on the Numbers, to 100 on the Segments, then againſt 8, the dry Inches on the Numbers, is 17.4 on the Segments.

2. Set 100 upon B, to 97.455, the Cask's whole Content upon A, then againſt 17.4 upon B, you have 16.905 upon A, and ſo many Gallons it will take to fill up the Cask; this ſubtracted from 97,455, the whole Content, there reſt 80.555, the Quantity of Liquor in the Cask; which may otherwiſe be found thus: Having ſet 32 the Bung-Diameter to 100 on the Segments, againſt 24 the wet Inches on the Numbers, you have 82.25 on the Segments. *Then,*

As 100 *upon* B, *is to* 97.455 *upon* A;
So is 82.25 *upon* B, *to* 80.555 *upon* A: the ſame as above.

Note, The Line of Segments on this Rule is made for a *Spheroid*, and is therefore more exact for all Bulging Casks than the Table of the Segments, which are proper only for a Cylinder.

The following Tables ſhew the Ullage of two ſeveral Casks in Gallons and Quarts: They were made by drawing out the Liquor by an exact Wine-Quart, and the dry Inches taken by an Inſtrument proper for that Purpoſe.

A Table ſhewing the Ullage of a Cask in Gallons and Quarts, the Length of the Cask is 37.3 Inches, the Bung-Diameter 26.5, the Head-Diameter 25, and the Content 100 Wine Gallons.

Dry Inches.	Gall. & Quarts.		Dry Inches.	Gall. & Quarts.		Dry Inches.	Gall. & Quarts.	
0.71		1	6.19	13	2	10.98	32	2
1.10		2	6.33	14	0	11.09	33	0
1 49		3	6 48	14	2	11.20	33	2
1.62	1	0	6.62	15	0	11.31	34	0
1.82	1	1	6.76	15	2	11.42	34	2
2.00	1	2	6.90	16	0	11.53	35	0
2.16	1	3	7.04	16	2	11.64	35	2
2.30	2	0	7.18	17	0	11.75	36	0
2.44	2	1	7.32	17	2	11.86	36	2
2 57	2	2	7.45	18	0	11.97	37	0
2.69	2	3	7.60	18	2	12.08	37	2
2.80	3	0	7.73	19	0	12.19	38	0
2 91	3	1	7 87	19	2	12.30	38	2
3.01	3	2	8 00	20	0	12.41	39	0
3.11	3	3	8.14	20	2	12.52	39	2
3 26	4	0	8 26	21	0	12.63	40	0
3.30	4	1	8 39	21	2	12.74	40	2
3.39	4	2	8.52	22	0	12.85	41	0
3.48	4	3	8.65	22	2	12 96	41	2
3.57	5	0	8.77	23	0	13.07	42	0
3 65	5	1	8 90	23	2	13.18	42	2
3.73	5	2	9 02	24	0	13.28	43	0
3.81	5	3	9 14	24	2	13.39	43	2
3 89	6	0	9 26	25	0	13 49	44	0
4 05	6	2	9.38	25	2	13.60	44	2
4.21	7	0	9.50	26	0	13.70	45	0
4.37	9	2	9 62	26	2	13.81	45	2
4.53	8	0	9 73	27	0	13 92	46	0
4 69	8	2	9.85	27	2	14 02	46	2
4 84	9	0	9.96	28	0	14 12	47	0
4 99	9	2	10.08	28	2	14.23	47	2
5.15	10	0	10 19	29	0	14 33	48	0
5.30	10	2	10.31	29	2	14.44	48	2
5.45	11	0	10 42	30	0	14.54	49	0
5.60	11	2	10.54	30	2	14.65	49	2
5.75	12	0	10.65	31	0	14.75	50	0
5.90	12	2	10.76	31	2			
6.04	13	0	10 87	32	0			

A Table ſhewing the Ullage of a Caſk to every half Gallon, the Length of the Caſk is 30.8 Inches, the Bung-Diameter 25.6, the Head-Diameter 22.6, and the Content 64 Wine-Gallons.

Dry Inch.	Gall. and Quart		Dry Inch.	Gall. and Quart		Dry Inch.	Gall. and Quart	
.80		2	5.76	11	2	9.60	22	2
1.25	1	0	5 94	12	0	9.77	23	0
1.60	1	2	6.12	12	2	9.94	23	2
1.91	2	0	6.30	13	0	10.11	24	0
2.17	2	2	6.48	13	2	10.28	24	2
2 40	3	0	6.66	14	0	10.45	25	0
2.63	3	2	6.84	14	2	10.62	25	2
2.85	4	0	7.01	15	0	10.79	26	0
3.07	4	2	7.19	15	2	10.96	26	2
3.28	5	0	7.36	16	0	11.13	27	0
3.49	5	2	7.54	16	2	11.30	27	2
3.69	6	0	7 71	17	0	11.47	28	0
3.89	6	2	7.89	17	2	11 64	28	2
4.08	7	0	8.06	18	0	11.81	29	0
4.27	7	2	8.24	18	2	11.98	29	2
4.46	8	0	8 41	19	0	12.14	30	0
4.65	8	2	8.58	19	2	12.31	30	2
4.84	9	0	8.75	20	0	12.47	31	0
5.03	9	2	8.92	20	2	12.64	31	2
5.21	10	0	9 09	21	0	12.80	32	0
5.40	10	2	9.26	21	2			
5.58	11	0	9.43	22	0			

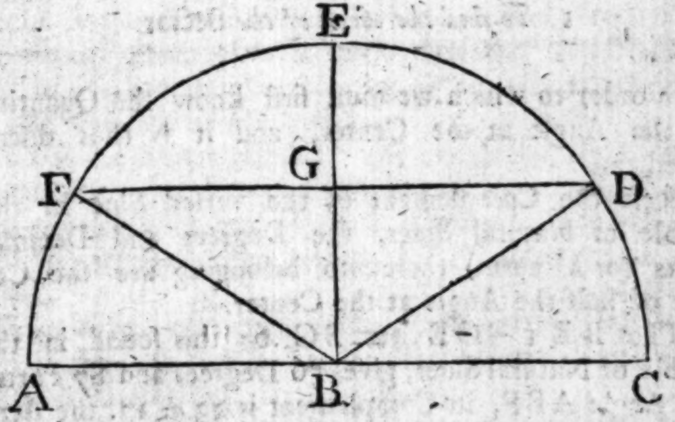
The Casks by which theſe Tables were made, were a Brandy Punchion, and a Brandy Hogſhead, both from *Bourdeaux*: The Tables will ſerve indifferently for all ſuch Casks, whoſe Diameters, Length, and Contents are the ſame with theſe, and are ſo plain, that they need no Example to ſhew their Uſe; for againſt the dry Inches you have the Gallons and Quarts, that the Cask wants of being full, and if the Bung-Diameter be more than half dry, enter the Table with the wet Inches inſtead of the dry, it ſhews the Quantity of Liquor in the Cask. Theſe Tables may by Proportion be converted into Lines and put upon a Rule.

By what hath been already ſaid, 'tis eaſy (I preſume) to find the Ullage of a Cask whoſe Axis is poſited Parallel to the Horizon: But leſt this Tract ſhould be thought imperfect, I ſhall here inſert a Table of the Segments of a Circle, and alſo ſhew their Conſtruction and Uſe.

A Table of the Segments of a Circle whoſe Area is Unity, the Diameter (*viz.* 1.128378) being divided into 100 equal parts.

V.S	Segm	V.S	Segm.	V.S	Segm.	V.S	Segm.
1	.0017	99	.9983	26	.2066	74	.7934
2	.0048	98	.9952	27	.2178	73	.7822
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7359
7	.0308	93	.9692	32	2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7124
9	0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	3119	65	6881
11	0598	89	.9402	36	3241	64	.6759
12	.0680	88	.9320	37	3364	63	.6636
13	.0764	87	.9236	38	.3487	62	.6513
14	0851	86	9149	39	.3611	61	.6389
15	.0941	85	.9059	40	.3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8879	42	.3986	58	.6014
18	.1224	82	8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	5762
20	.1424	80	.8576	45	4364	55	.5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1621	78	.8366	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Conſtruſtion *of the* Table *of* Segments *of a* Circle.



In the Figure here before us, all that Part or Proportion comprehended within the right Lines D B and B F, and the Circular Line F E D, is called the Sector of a Circle. F G D is the Chord of the Segment, G E is the Verſed Sine, and G B the Complement thereof, A B C is the Diameter of the Circle, which here we ſuppoſe to be 2.

Now, if the Diameter of a Circle be 2; the Length of the Circumference is double the Area of the Circle; for the Circumſtance is 6.2831852, and the Area 3.1415926. Hence it evidently follows, that the Length of half the Arch of any Sector of this Circle, is equal to the Area of that Sector and (as ſhall be proved anon) the natural Sine of the ſaid half Arch is equal to the Area of the Triangle in that Sector.

Now, the Area of the Triangle, taken from the Area of the Sector, leaves the Area of the Segment.

Example. Let it be required to find the Area of the Segment FEDG, whoſe verſed Sine GE is .4.

1. *To find the Area of the* Sector.

In order to which we muſt firſt know the Quantity of the Angle at the Center, and it is thus diſcovered.

Seek the Complement of the verſed Sine in the Table of Natural Sines, the Degrees and Decimal Parts (or Minutes) thereunto belonging are the Coſine of half the Angle at the Center.

Thus BE 1.—GE .4 = BG .6; this found in the Table of Natural Sines, gives 56 Degrees and 87 Parts; the Angle ABF, its Complement is 53 *d.* 13. the Arch FE; which doubled, is 106 *d.* 36 the Arch of the Sector FED; that is, the Quantity of the Angle at the Center.

Now to find the Length of any Number of Degrees, (that is the Length of any Arch) of this Circle, ſay,

As 360, *the Degrees in the whole Circle, to* 6.2831852,
the Length of the whole Circumference;
So *is* 1, *to* .01745329, *the Length of one Degree.*

And this is a common Multiplicator for any Number of Degrees; ſo the Degrees in the Arch FE, *viz.* 53. 13, being multiplied by .01743529, gives .9272932977, the Length of the Arch FE; that is, the Area of the Sector BFED, which was required.

2. *To find the Area of the Triangle* B F D.

The Diameter being 2, I ſay, half the Natural Sine of the Angle at the Center, when the ſaid Angle does not exceed 90 *d*, when it does, then half the Natural Sine of its Complement to 180, is equal to the Area of the Triangle in the Sector.

Thus the Sector B F E D, the Angle of the Center is 106.26; its Complement to 180 is 73.74, whoſe Natural Sine by the Table is .96 the half of this is .48, the Area of the Triangle B F D, which was required.

The Truth of which doth thus appear; B F is 1, B G is .6; *ergo* G F is .8; For B F q — B G q = F G q (by 47. 1 of *Euclid.*)

But FG .8 multiplied by B G .6, gives .48 the Area of the Triangle B E D, as before. Then,

From the Area of the Sector B F E D	.92729329
Take the Area of the Triangle B F D	.48000000
There remains the Area of the Segment F E D G	.44729329

But ſeeing in all Tables ſhewing the Area's of the Segments of a Circle, it is ſuppoſed the Area of the whole Circle is Unity, the foregoing Table was made by this Proportion:

As the Area of that Circle whoſe Diameter is 2, *viz.* 3.1415926, is to the Segment of any Part of that Diameter;

So is 1 (the ſuppoſed Area of another Circle) to the Segment of the like Part of its Diameter. Thus,

As 3.1415926, is to .49729329, the Segment laſt found;

So is 1 to .14237, the Segment anſwering to the verſed Sine 20, in the Table.

For as the verſed Sine of the firſt Segment, *viz.* G E, was .4, that is, $\frac{2}{3}$ of the Radius B E;

So 20, the verſed Sine of this laſt Segment is $\frac{2}{3}$ of the Radius of the Table.

The Uſe of the Table *of* Segments,

Is, to find the Ullage or Quantity of Liquor remaining in a Cask, whoſe Axis is poſited parallel to the Horizon the Surface of the Liquor cutting the Heads of the Cask; and for this the Rule is,

To the wet or dry Inches of the Bung-Diameter add a competent Number of Cyphers; then divide it by the whole Diameter; the Quotient found in the Table, under the Title *V. S,* gives a Segment, which multiplied by the whole Content of the Cask, the *Product* ſhews the Quantity of Liquor in the Cask, if the Dividend was the wet Inches; or the Ullage, if it was the dry.

Example. Let there be a Cask in the Form of a Cylinder, whoſe Bung-Diameter is 432 Inches, the dry part 12, and the Content of the Cask 108 Gallons: How many Gallons are wanting to fill up the Cask?

432 divided by 12, quotes 36; which found in the Table under *V. S.* gives .3241; this Segment multiplied by 108 (the whole Content of the Cask) the Product is 35; and ſo many Gallons the Cask wants of being full.

Note, If a Cask be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage more exactly than the Line of Segments on the Rule:

but if the Bung-Diameter be much greater than the Head-Diameter, the Line of Segments is truer than the Table.

PROB. VII. Fig. 23. Plate III.

There is a Cask ſuppoſed to be the middle Fruſtum of a Spheroid, ſtanding upon one Head, with its Axis perpendicular to the Horizon; the Dimenſions

Are	E H	Length ——	24	Inches.
	A B	Bung-Diameter ——	20	
	C D	Head-Diameter ——	26	

To calculate any Diameter betwixt the Bung and Head (ſuppoſe S V) and by ſuch Diameter to find the Quantity of Liquor in the Cask, when part is empty.

1. *To find the Diameter, the Rule is,*

From the Square of the Bung-Diameter (A B) ſubtract the Square of the Head-Diameter (C D) and divide the Square Root of the remainder by Half the Cask's Length (E o); then multiply this Quotient by the Number of Inches that the Diameter ſought is diſtant from the Bung, and call this Product your Subducend. Laſtly, from the Square of the Bung-Diameter, ſubtract the Square of your Subducend: The Square Root of the Remainder is the Diameter ſought.

Example. Let it be required to find the Diameter S V, which is ſuppoſed to be 6 Inches diſtant from A B, the Bung-Diameter.

The Square of the	Bung-Diameter is	400
	Head-Diameter is	256
The Remainder is		144

W[illegible]quare Root is 12; this divided by half the [illegible]k's Length, *viz.* 12, the Quotient is 1; this multiplied by 6, gives 6 for the Subducend,

Square of the Bung-Diameter	400
Square of the Subducend	36
There remains	364

Which is the Square of the Diameter S V; its Square Root is 19.07878, the Diameter itſelf: But the Square of it is ſufficient for our Pupoſe.

2. *To find the Quantity of Liquor in the Caſk.*

Having found the Square of S V, the Diameter of the Liquor's Surface, the Quantity of Liquor is eaſily attained.

For by the Rule laid down in the firſt Problem of this Section, the whole Content of the Cask will be found to be 23.532 Ale-Gallons; the half of this is 11.766, for the Content of the lower half A n t B, and the Part A S V B will be found to be 6.484 Gallons.

For the Square of A B is ——————— 400
The Square of S V is ——————— 364

Sum is ——————— 764
Half Sum is ——————— 382
Half the Difference of the Squares is ——— 18

Sum of theſe is ——————— 1164

This multiplied by *o g*, *viz.* 6, gives 6984; and this divided by 1077, quotes 6.484, the Content of the Part A S V B, as above: Now this added to the Content of the lower half, *viz.* 11.766, gives 18.25; and ſo many Gallons are in the Caſk. If this be taken from 23.532, the whole Content, there reſts 5.282; and ſo many Gallons are required to fill up the Caſk: For this 5.282 Gallons, is the Content of the Part S C D V.

If the Caſk were leſs than half full, the Ullage might be found by the ſame Method; and that ſubtracted from the whole Content, leaves the Quantity of Liquor in the Caſk.

SECT. X.

Of Gauging of Worts *in open Veſſels.*

FOR this Purpoſe, there are two Lines placed on the underſide of the two ſliding Pieces of the Rule; (ſee Plate I.) which being ſo plain and obvious, as not to need any Deſcription, I ſhall proceed to ſhew their Uſe, which is to find how many Ale-Gallons, and Hundredth Parts of a Gallon, any ſmall Tub, or ſuch like open Veſſel, (from 12 to 36 Inches Diameter) will contain at one Inch deep; to perform which, obſerve the following Directions.

☞ Let the Sliding Piece C D be pinn'd faſt at the End towards the Right Hand; then draw out the other ſliding Piece, till the two Ends touch any two oppoſite ſides of the Tub or Veſſel; then look what Diviſions (both upon the Line of Gallons and Line of Inches upon the ſliding Piece) are cut by the End of the Rule; for that Point ſhews the Diameter of the Tub in Inches and tenth Parts upon one Line; and upon the other, how many Gallons, and hundred Parts it will contain, at one Inch deep.

Thus, when the End of the Rule cuts 13.4 on the Line of Inches, it alſo cuts 0 Gallons, .5 Tenths on the Line of Gallons. In like manner, when it cuts 21 on the Inch Line, it will be at the ſame time cut 1.23 on the Line of Gallons; that is, 1 Gallon, and .23 hundredth Parts; and ſo much will a Tub of 21 Inches Diameter contain at one Inch deep. And thus you may draw out this Piece to 24 Inches and an half, and the End of the Rule will cut 1.673 on the Line of Gallons.

Now, if the Diameter of the Tub be more than 24.5, put this Piece at 24.5, and then draw out the other Sliding Piece at the other End, where you will find, that when the End of the Rule cuts 30 Inches, it will alſo cut 2.51 Gallons; and thus you will find 3 Gallons, 6 Tenths and more. *Note*, In all theſe Caſes 'tis the Area only which is ſought; the Inches alſo ſhew the Diameter; but this is to make the other more plain.

Now, the Tubs or Veſſels which Victuallers commonly make uſe of for cooling their Worts, and working their Ale, are generally Cylindrical or Conical, and in either of theſe the Baſes are Circular or Elliptical.

Firſt, Of ſuch as are Cylinders, or near that Form.

If the Baſes of a Cylindrical Tub be circular, the Rule ſhews the Area or Content at one Inch deep by Inſpection (as above;) this multiplied by the Depth of the Liquor, gives the Content.

For Inſtance.

Admit I come to a Tub, and (by the Rule) find the Area to be 1 Gallon 4 Tenths, and the Depth of the Wort 7 Inches and an half.

Multiply the Area ——————— 1.4
by the Depth, *viz.* ——————— 7.5

70
.98

The Product is ——————— 10.50

That is, 10 Gallons and an half, which is the Content ſought.

But this Content may be found without the Pen; for having the Area and the Depth, you may multiply the one by the other, by the Rule: Thus,

Set 1 upon B, to 1.4, the Area, upon A; then againſt 7.5, the Depth, upon B, you have 10.5 upon A, the Content as before.

If the Veſſel be Elliptical, take the croſs Diameters in the Middle of the Depth; as, ſuppoſe the longeſt Diameter is 60 Inches, and the ſhorteſt 40; ſay,

As 359 is to one of the Diameters;
So is the other Diameter to the Area.

And by this Proportion on the Lines A and B, the Area of the Ellipſis above-mentioned will be found to be 6.68: For,

As 359 upon B, is to 40 upon A;
So is 60 upon B, to 6.68 upon A.

This is the true Way of finding the Area of an Ellipſis. But in ſmall Elliptical Tubs you may find the Area with leſs Trouble: Thus,

Find (by drawing out the Rule) the Area of the longeſt Diameter; and in like manner, the Area of the ſhorteſt Diameter; half the Sum of theſe two is the Area ſought.

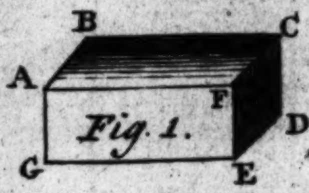


Fig. 1.

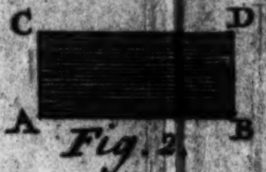
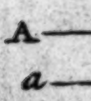
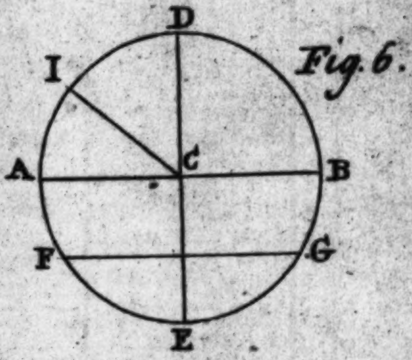


Fig. 2.

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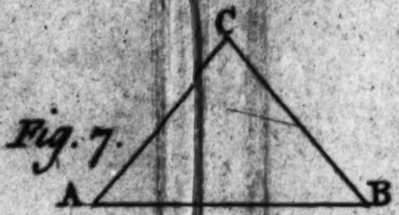
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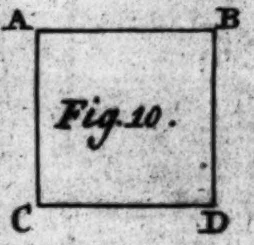
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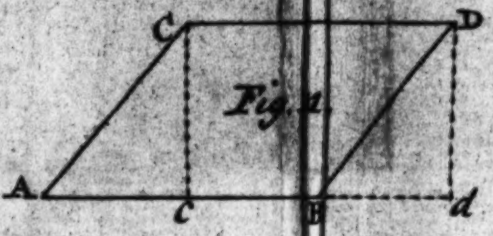
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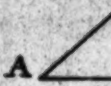

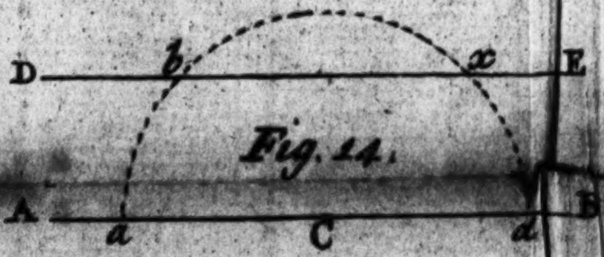
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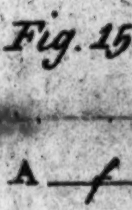


Fig. 15

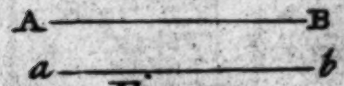
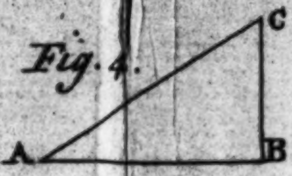


Fig. 3.



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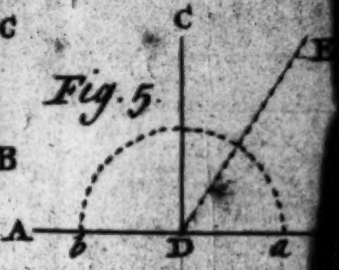

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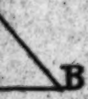

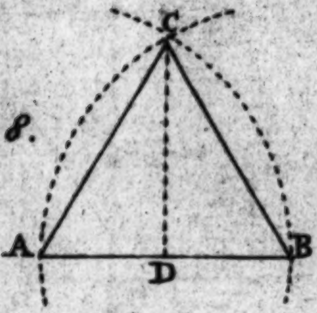

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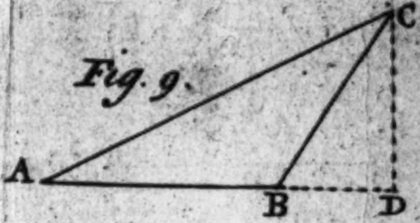

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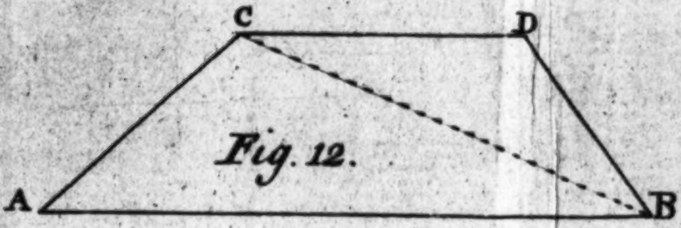

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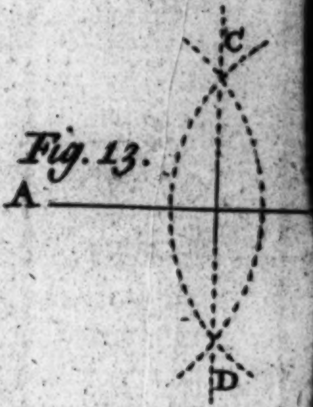

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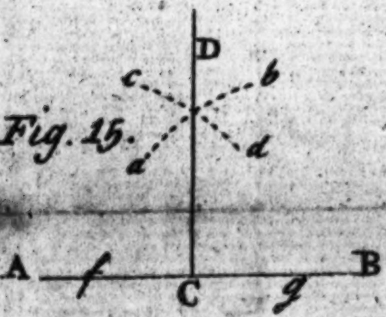

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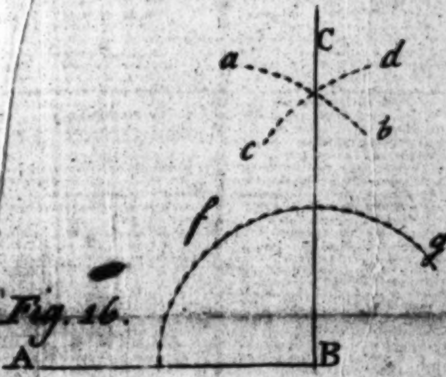

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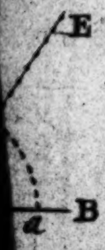

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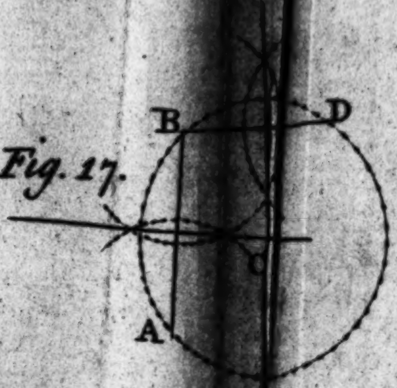

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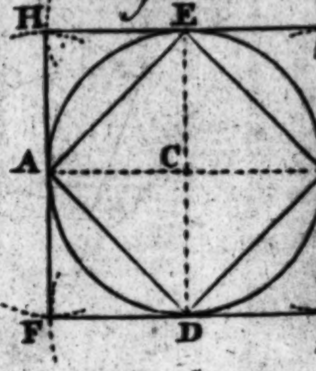

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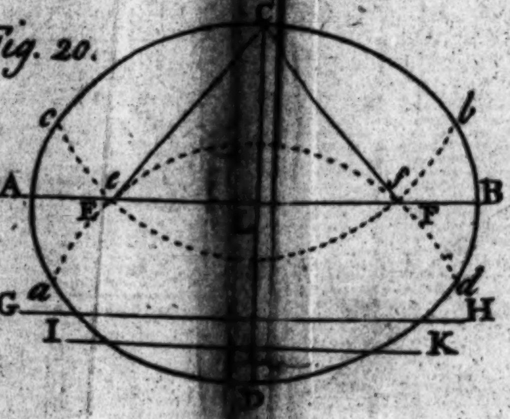

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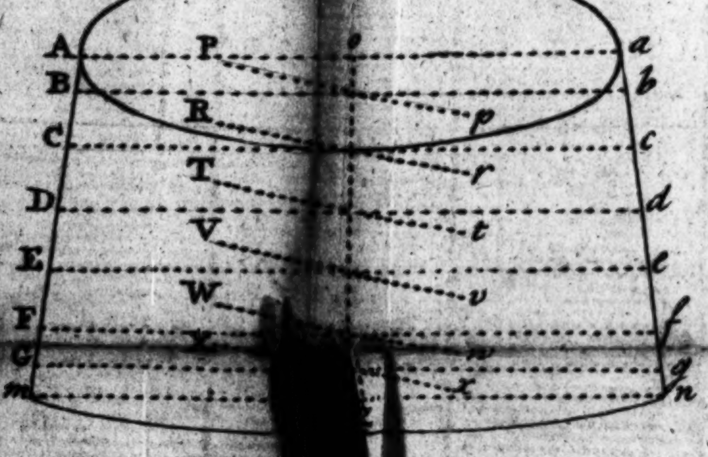

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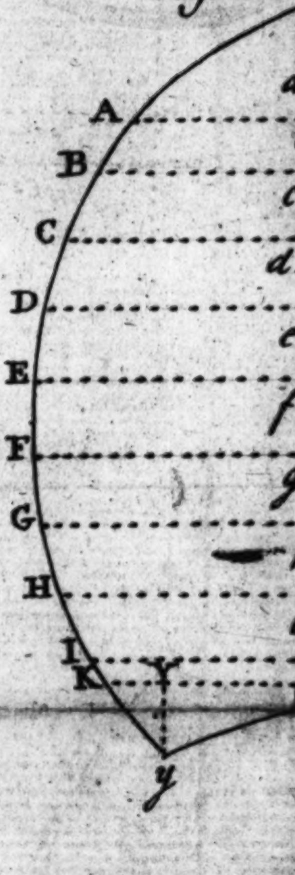
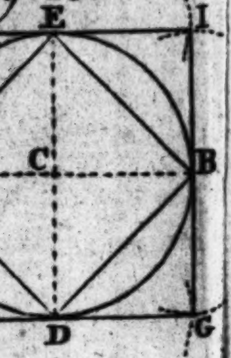


Plate II. *Facing Pag. 144.*

ig. 18.



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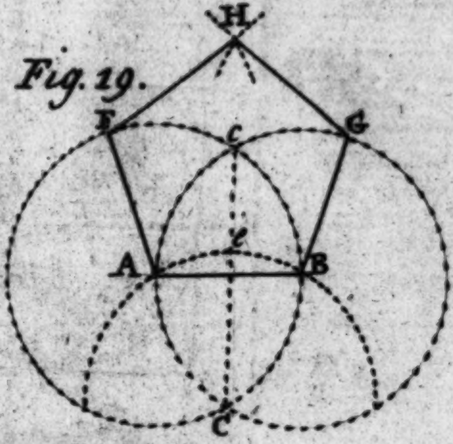

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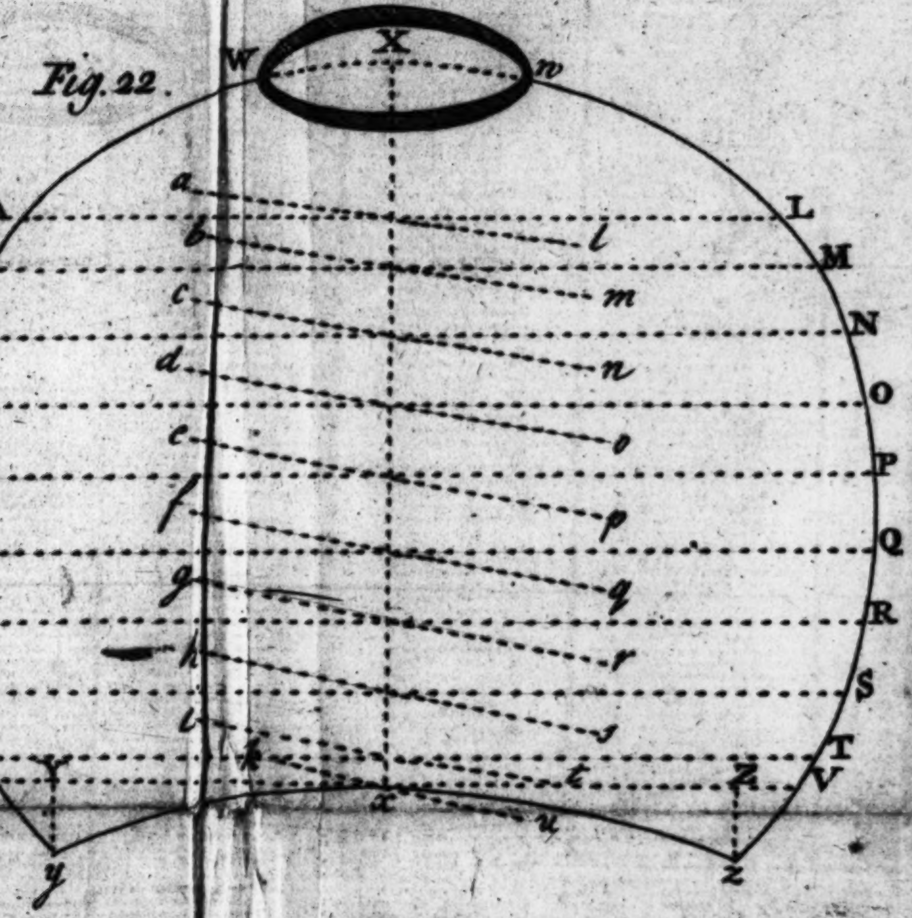

For Inſtance.

Suppoſe the Area of the longeſt Diameter be 3.62
And the Area of the ſhorteſt ———— 2.51
The Sum is ———————— 6.13
The half is ———————— 3.065

The Area required, which is very near the Truth: For if you work by the former Rule, you will find the true Area to be 3.008.

Secondly, for Conical Veſſels.

When the Diameters at Top and Bottom differ not above four Inches, you may find the Area in the middle of the Depth, by drawing out the Rule, as above directed; and this Area thus found, is very near a true Mean for the whole Tub.

Example. Let the Diameter at the Bottom of a Conical Tub be 24 Inches, and the Diameter at the Top 28 Inches; in the Middle of the Depth of this Tub you will find (by the Rule) the Diameter 26 Inches, and the Area 1.88 Gallons.

And for ſuch Conical Tubs (or Tuns) whoſe Diameters are above 36 Inches, if they be near a Cylinder, one mean Inch will be ſufficient.

But for ſuch as are very Conical and great, you may find the whole Content by Prob. V. Sect. VII. And if you deſire to inch any ſuch Tun, obſerve the Directions in Prob. I. Sect. VIII.

There are yet another Sort of Veſſels which Victuallers frequently uſe to work their Ale in; and theſe are Part of a Butt or Pipe ſet upon one Head, the others being cut off.

The whole Content of theſe may be found by Prob. VII. Sect. IX.

I have obſerved in ſeveral Places, they cool their Worts in Braſs-Pans, whoſe Diameter is about 30 Inches, and Depth 10 or 12 Inches; but theſe are commonly deeper in the middle by an Inch or two than at the Side.

In ſuch Veſſels as theſe, if you take (with the Rule) the Area of the Liquor's Surface, and multiply it by the Depth taken within 3 or 4 Inches of the Side, the Product will be the Content very near the Truth.

And for round Bowls, which are like the Segment of a Globe, if you multiply the Area of the Liquor's Surface by half the Depth, it gives the Content near enough in Practice: But in ſuch Caſes as theſe, Experience will be your beſt Tutor.

But, that I may not be wanting to the meaneſt Capacities, I have here inſerted a very uſeful Table, ſhewing the Area's of Circles and Contents of Cylinders in Ale-Gallons.

THE EXPLANATION and USE OF THE

Table of Cylinders.

THE Table conſiſts of two Parts: The Firſt ſhews the Area's of Circles, and Contents of Cylinders, from 12 to 42 Inches diameter, and from 1 to 10 Inches deep, (and by Addition, to any other Depth whatſoever.) The ſecond Part ſhews only the Area's of Circles, from 42 to 156 Inches Diameter.

In the firſt Part of the Table, the two firſt Pages (and ſo every two next ſucceeding) contain 10 Columns: In the firſt of which you have the Diameters of Circles beginning at 12, and continued down (in the ſaid firſt Column on every left-hand Page) in Inches and Tenths, to 42 Inches.

The other Nine Columns are numbered at the Top, 1, 2, 3, 4, *&c.* to 9. In the the firſt of theſe, under 1, you have (againſt their reſpective Diameters) the Area's of Circles, (or the Content at one Inch deep) to the hundred thouſandth-part of an Ale-Gallon.

In the next Column, under 2, is the Content at 2 Inches deep; under 3, the Content at 3 Inches deep; and ſo on to 9 Inches deep in the laſt Column under 9.

Thus, if the Diameter of a Circle be 29.6 Inches, the Area is 2.4401 Gallons, the Content at 1 Inch deep. And if the Diameter of a Cylinder by 29.6 Inches.

The Content.	2	Inches deep is	4.88	Gallons.
	3		7.32	
	4		9.76	
	5		12.20	
	6		14.64	
	7		17.08	
	8		19.52	
	9		21.96	

Note, If the former Cylinder of 29.6 Inches Diameter,

Be	10	Inches deep, the Content will be	24.4	Gallons.
	20		48.8	
	30		73.2	
	40		97.6	
	50		122.0	
	60		146.4	
	70		170.8	
	80		195.2	
	90		219.6	

For the Content of any Cylinder at 2 Inches, and 20 Inches deep, is expreſſed by the ſame Figures: But as 20 is ten times 2, the Area at 2 Inches deep muſt be made ten times greater; which is done by only putting the Prick, or Point one Place more towards the Right-hand. From hence the Uſe of this Table is obvious, as may further appear by the following Example.

There is a Tun in the Form of a Cylinder, the Diameter is 29.6, *and the Depth* 27.8 *Inches. What is the Content in Gallons?*

At 29.6 Inches Diameter,

20 Inches deep is	48.80
7 Inches deep is	17.08
.8 of an Inch deep is	1.95
27.8 Content ſought is	67.83

If the mean Diameter of your Tun be above 52 Inches, find it in the firſt Column of the ſecond Part of the Table: For againſt the whole Inches of your Diameter in the Side of the Table, and under the Tenths (if any be) you have the Area or Content at one Inch deep; this multiplied by the whole Depth, gives the Content of the Tun. But this is ſo plain, it needs no Example.

And for Caſks, find the mean Diameter by the Direction given in Page 127; and then by the Length and mean Diameter the Content is found as above.

So, if the Length of a Caſk be 30 Inches, and the mean Diameter 23 Inches, the Content will be found to be 44 2 Gallons: For againſt 3, the mean Diameter, and under 3, is 4.42. Now, if 3 be 30, 4.42 will be 44 2, the Content ſought.

A
TABLE
OF
CYLINDERS,

From 12, to 42 Inches Diameter;
and from 1, to 9 Inches Deep:

Very Uſeful in Gauging of

SMALL TUNS and CASKS.

Shewing by Inſpection,

The CONTENT of ſuch Veſſels to the Thouſandth Part of a Gallon, Ale or Wine-Meaſure.

Diam. in Inches	DEPTH.			
	1	2	3	4
12.0	0.4011	0.80	1.20	1.60
.1	0.4078	0.81	1.22	1.63
.2	0.4146	0.83	1.24	1.66
.3	0.4214	0.84	1.26	1.69
.4	0.4282	0.86	1.28	1.71
.5	0.4352	0.87	1.30	1.74
.6	0.4422	0.88	1.33	1.77
.7	0.4492	0.90	1.35	1.80
.8	0.4563	0.91	1.37	1.82
.9	0.4635	0.93	1.39	1.85
13.0	0.4707	0.94	1.41	1.88
.1	0.4780	0.96	1.43	1.91
.2	0.4853	0.97	1.46	1.94
.3	0.4927	0.98	1.48	1.97
.4	0.5001	1.00	1.50	2.00
.5	0.5076	1.01	1.52	2.03
.6	0.5151	1.03	1.55	2.06
.7	0.5227	1.04	1.57	2.09
.8	0.5304	1.06	1.59	2.12
.9	0.5381	1.08	1.61	2.15
14.0	0.5459	1.09	1.64	2.18
.1	0.5537	1.11	1.65	2.21
.2	0.5616	1.12	1.68	2.24
.3	0.5695	1.14	1.71	2.28
.4	0.5775	1.15	1.73	2.31
.5	0.5856	1.17	1.76	2.34
.6	0.5937	1.19	1.78	2.37
.7	0.6018	1.20	1.80	2.40
.8	0.6100	1.22	1.83	2.44
.9	0.6183	1.24	1.85	2.47

DEPTH.				
5	6	7	8	9
2.01	2.41	2.81	3.21	3.61
2.04	2.44	2.85	3.26	3.67
2.07	2.49	2.90	3.31	3.73
2.11	2.53	2.95	3.37	3.79
2.14	2.57	3.00	3.43	3.86
2.18	2.61	3.05	3.48	3.92
2.21	2.65	3.10	3.54	3.98
2.24	2.69	3.14	3.59	4.04
2.28	2.73	3.19	3.65	4.10
2.32	2.78	3.24	3.71	4.17
2.35	2.82	3.30	3.77	4.24
2.39	2.87	3.35	3.82	4.30
2.43	2.91	3.39	3.88	4.36
2.46	2.95	3.44	3.94	4.43
2.50	3.00	3.50	4.00	4.50
2.53	3.04	3.55	4.06	4.56
2.58	3.09	3.61	4.12	4.63
2.61	3.13	3.65	4.18	4.70
2.65	3.18	3.71	4.24	4.77
2.69	3.23	3.77	4.30	4.84
2.73	3.27	3.82	4.36	4.91
2.76	3.32	3.87	4.42	4.98
2.81	3.37	3.93	4.49	5.05
2.84	3.41	3.98	4.55	5.12
2.88	3.46	4.04	4.62	5.19
2.93	3.51	4.10	4.68	5.26
2.96	3.56	4.15	4.74	5.34
3.01	3.61	4.21	4.81	5.41
3.05	3.66	4.27	4.88	5.49
3.09	3.71	4.33	4.94	5.56

Diam. in Inches	DEPTH.			
	1	1	4	4
15.0	0.6266	1.26	1.88	2.50
.1	0.6350	1.27	1.91	2.53
.2	0.6435	1.29	1.93	2.57
.3	0.6520	1.30	1.95	2.61
.4	0.6604	1.32	1.98	2.64
.5	0.6691	1.34	2.01	2.68
.6	0.6778	1.35	2.03	2.71
.7	0.6865	1.37	2.06	2.74
.8	0.6953	1.39	2.09	2.78
.9	0.7041	1.41	2.11	2.82
16.0	0.7130	1.43	2.14	2.85
.1	0.7219	1.44	2.16	2.89
.2	0.7309	1.46	2.19	2.92
.3	0.7400	1.48	2.22	2.96
.4	0.7491	1.50	2.25	3.00
.5	0.7582	1.52	2.27	3.03
.6	0.7654	1.53	2.30	3.07
.7	0.7767	1.55	2.33	3.10
.8	0.7861	1.57	2.36	3.14
.9	0.7954	1.59	2.39	3.18
17.0	0.8049	1.61	2.41	3.22
.1	0.8144	1.63	2.44	3.26
.2	0.8239	1.64	2.47	3.29
.3	0.8335	1.67	2.50	3.33
.4	0.8432	1.69	2.53	3.37
.5	0.8529	1.70	2.56	3.41
.6	0.8627	1.72	2.58	3.45
.7	0.8725	1.74	2.62	3.49
.8	0.8824	1.76	2.65	3.53
.9	0.8924	1.78	2.68	3.57

DEPTH.				
5	6	7	8	9
3.13	3.76	4.38	5.01	5.63
3.18	3.81	4.45	5.08	5.72
3.22	3.86	4.50	5.15	5.79
3.26	3.91	4.56	5.22	5.87
3.30	3.96	4.62	5.28	5.94
3.35	4.01	4.68	5.35	6.02
3.39	4.06	4.74	5.42	6.09
3.43	4.12	4.80	5.49	6.18
3.48	4.17	4.87	5.56	6.26
3.52	4.22	4.93	5.63	6.34
3.57	4.28	5.99	5.70	6.42
3.61	4.33	5.05	5.78	6.50
3.65	4.39	5.12	5.85	6.58
3.70	4.44	5.18	5.92	6.66
3.75	4.49	5.24	5.99	6.74
3.79	4.53	5.31	6.06	6.82
3.84	4.60	5.37	6.14	6.90
3.88	4.66	5.43	6.21	6.99
3.93	4.71	5.50	6.28	7.07
3.98	4.77	5.57	6.36	7.16
4.02	4.83	5.63	6.44	7.24
4.07	4.88	5.70	6.51	7.33
4.12	4.94	5.77	6.59	7.42
4.17	5.00	5.83	6.66	7.50
4.22	5.06	5.90	6.74	7.59
4.26	5.11	5.97	6.82	7.68
4.31	5.17	6.03	6.90	7.76
4.36	5.23	6.10	6.98	7.85
4.41	5.29	6.17	7.06	7.94
4.46	5.35	6.24	7.14	8.03

Diam. in Inches.	DEPTH.			
	1	2	3	4
18.0	0.9024	1.80	2.71	3.61
.1	0.9124	1.82	2.74	3.65
.2	0 9225	1.84	2.77	3.69
.3	0.9327	1.87	2.80	3.73
.4	0 9429	1.88	2.83	3.77
.5	0 9532	1.91	2.86	3.81
.6	0.9635	1.93	2.89	3.85
.7	0.9739	1.95	2.92	3.90
.8	0.9843	1.97	2.95	3.94
.9	0.9948	1.99	2.98	3.98
19.0	1.0054	2.01	3.02	4.02
.1	1.0160	2.03	3 05	4.06
.2	1.0267	2.05	3.08	4.11
.3	1.0374	2.07	3.11	4 15
.4	1.0482	2.10	3.14	4.19
.5	1.0590	2.12	3.18	4 24
.6	1.0699	2.14	3.21	4.28
.7	1.0808	2.16	3.24	4.32
.8	1.0918	2.18	3.27	4.37
.9	1.1020	2.21	3.31	4.41
20.0	1.1140	2 23	3.34	4.46
.1	1.1252	2.25	3.38	4.50
.2	1.1364	2.27	3 41	4.55
.3	1.1477	2 29	3.44	4 59
.4	1.1590	2.32	3.48	4.64
.5	1.1704	2.34	3.51	4.68
.6	1.1819	2.36	3 54	4.73
.7	1 1934	2.39	3.58	4.77
.8	1.2049	2.41	3.61	4.82
.9	1.2165	2.43	3 65	4.86

DEPTH.				
5	6	7	8	9
4.51	5.41	6.31	7.22	8.12
4.56	5.47	6.38	7.30	8.21
4.61	5.53	6.45	7.38	8.30
4.66	5.59	6.53	7.46	8.39
4.71	5.66	6.60	7.54	8.49
4.77	5.72	6.67	7.63	8.58
4.82	5.78	6.74	7.71	8.67
4.87	5.85	6.82	7.79	8.77
4.92	5.90	6.89	7.87	8.86
4.97	5.97	6.96	7.95	8.95
5.03	6.03	7.04	8.04	9.05
5.08	6.10	7.11	8.13	9.14
5.13	6.16	7.18	8.21	9.23
5.19	6.22	7.26	8.30	9.34
5.24	6.29	7.34	8.38	9.43
5.30	6.35	7.41	8.47	9.53
5.35	6.42	7.49	8.56	9.63
5.40	6.49	7.57	8.65	9.73
5.46	6.55	7.64	8.74	9.83
5.51	6.62	7.72	8.82	9.93
5.57	6.68	7.80	8.91	10.03
5.63	6.75	7.88	9.00	10.13
5.68	6.82	7.95	9.09	10.23
5.74	6.88	8.03	9.18	10.33
5.80	6.95	8.11	9.27	10.43
5.85	7.02	8.19	9.36	10.53
5.91	7.09	8.27	9.46	10.64
5.97	7.16	8.35	9.54	10.74
6.02	7.23	8.43	9.64	10.84
6.08	7.30	8.51	9.73	10.94

Diam. in Inches.	DEPTH.			
	1	2	3	4
21,0	1,2282	2,46	3,68	4,91
,1	1,2400	2,48	3,72	4,96
,2	1,2517	2,50	3,75	5,01
,3	1,2635	2,53	3,79	5,05
,4	1,2754	2,55	3,83	5,10
,5	1,2874	2,57	3,86	5,15
,6	1,2994	2,60	3,90	5,20
,7	1,3114	2,62	3,93	5,25
,8	1,3236	2,65	3,97	5,29
,9	1,3357	2,67	4,01	5,34
22,0	1,3480	2,70	4,04	5,39
,1	1,3602	2,72	4,08	5,44
,2	1,3726	2,75	4,12	5,49
,3	1,3850	2,77	4,16	5,54
,4	1,3974	2,79	4,19	5,59
,5	1,4099	2,82	4,23	5,64
,6	1,4225	2,84	4,27	5,69
,7	1,4351	2,87	4,31	5,74
,8	1,4478	2,90	4,34	5,79
,9	1,4605	2,92	4,38	5,84
23,0	1,4733	2,97	4,42	5,89
,1	1,4861	2,99	4,46	5,94
,2	1,4990	3,00	4.50	6,01
,3	1,5120	3,02	4,54	6,05
,4	1,5250	3,05	4,58	6,10
,5	1,5380	3,08	4,61	6,15
,6	1,5511	3,10	4,65	6,20
,7	1,5643	3,13	4,69	6,26
,8	1,5775	3,15	4,73	6,31
,9	1,5908	3,18	4,77	6,36

DEPTH.				
5	6	7	8	9
6,14	7,37	8,60	9.82	11,05
6,20	7,44	8,68	9.92	11,16
6,26	7,51	8,76	10,01	11,26
6,32	7,58	8,84	10,11	11,37
6,38	7,65	8,93	10,20	11,48
6,44	7,72	9,01	10,30	11,59
6,50	7,80	9,09	10,39	11,69
6,56	7,87	9,18	10,49	11,80
6,62	7,94	9,26	10,59	11,91
6,68	8,01	9,35	10,68	12,02
6,74	8,09	9,44	10,78	12,13
6,80	8,16	9,52	10,88	12,24
6,86	8,23	9.61	10,98	12,35
6,93	8,31	9,70	11,08	12,47
6,99	8,38	9,78	11,18	12,58
7,05	8,46	9,87	11,28	12,69
7,11	8,53	9 95	11,38	12,80
7,18	8,61	10,05	11,48	12,92
7,24	8,68	10,13	11,58	13,02
7,30	8,76	10,22	11,68	13,14
7,37	8,84	10,31	11,78	13,26
7,43	8,92	10,40	11,89	13,37
7,50	8,99	10,49	11,99	13,49
7,56	9,07	10,58	12,10	13,61
7,63	9,15	10,68	12,20	13,73
7,69	9,23	10,77	12 30	13,84
7,76	9,31	10,86	12,41	13,96
7 82	9,39	10,95	12,51	14,08
7,89	9,46	11,04	12,62	14,23
7,95	9,54	11,13	12,72	14,31

Diam. in Inches.	DEPTH.			
	1	2	3	4
24,0	1,6042	3,21	4,81	6,42
,1	1,6176	3,23	4,85	6,47
,2	1,6310	3,26	4,89	6,52
,3	1,6445	3,29	4,93	6,58
,4	1,6581	3,32	4,97	6,63
,5	1,6717	3,34	5,01	6,69
,6	1,6854	3,37	5,09	6,74
,7	1,6991	3,40	5,10	6,80
,8	1,7129	3,43	5,14	6,85
,9	1,7267	3,45	5,18	6,91
25,0	1,7406	3,48	5,22	6,96
,1	1,7546	3,51	5,26	7,02
,2	1,7686	3,54	5,31	7,07
,3	1,7827	3,57	5,35	7,13
,4	1,7968	3,59	5,39	7,19
,5	1,8110	3,62	5,43	7,24
,6	1,8252	3,65	5.48	7,30
,7	1,8395	3,68	5,52	7,36
,8	1,8538	3.71	5,56	7,42
,9	1,8682	3,74	5,60	7,47
26,0	1,8827	3,77	5,65	7,53
,1	1,8972	3,79	5,69	7,59
,2	1,9117	3,82	5 73	7,65
,3	1,9264	3,85	5,78	7,71
,4	1,9411	3,88	5,82	7,76
,5	1,9558	3,91	5,87	7,82
,6	1,9706	3,94	5,91	7,88
,7	1,9854	3,97	5,96	7,94
,8	2,0003	4,00	6,00	8,00
,9	2,0153	4,03	6,05	8,06

DEPTH,				
5	6	7	8.	9
8,02	9,62	11,23	12,83	14,44
8,09	9,70	11,32	12,93	14,55
8,16	9,79	11,42	13,05	14,68
8,22	9,86	11,51	13,15	14,80
8,29	9,95	11,61	13,26	14,92
8,36	10,03	11,70	13,37	15,04
8,43	10,11	11,80	13,48	15,17
8,50	10,19	11,89	13,59	15,29
8,56	10,28	11,99	13,70	15,42
8,63	10,36	12,08	13,81	15,54
8,70	10,44	13,18	13,92	15,67
8,77	10,53	12,28	14,03	15,79
8,84	10,61	12,38	14,15	15,91
8,91	10,70	12,47	14,26	16,04
8,98	10,78	12,57	14,37	16,17
9,06	10,87	12,68	14,49	16,30
9,13	10,95	12,78	14,60	16,43
9,20	11,04	12,88	14,71	16,55
9,27	11,12	12,98	14,83	16,64
9,34	11,21	13,08	14,94	16,81
9,41	11,29	13,18	15,06	16,94
9,49	11,38	13,28	15,18	17,07
9,56	11,47	13,38	15,29	17,20
9,63	11,56	13,48	15,41	17,34
9,70	11,65	13,59	15,53	17,47
9,78	11,73	13,69	15,65	17,60
9,85	11,82	13,79	15,76	17,74
9,93	11,91	13,90	15,88	17,87
10,00	12,00	14,00	16,00	18,00
10,08	12,09	14,11	16,12	18,14

Diam. in Inches.	DEPTH.			
	1	2	3	4
27,0	2,0303	4,06	6.09	8,12
,1	2,0454	4,09	6,14	8,18
,2	2,0605	4,12	6,18	8,24
,3	2,0757	4,15	6,23	8,30
,4	2,0909	4,18	6,27	8,36
,5	2,1062	4,21	6,32	8,42
,6	2,1215	4,24	6,36	8,48
,7	2,1369	4,27	6,41	8,55
,8	2,1524	4,30	6,46	8,61
,9	2,1679	4,33	6,50	8,67
28,0	2,1835	4,37	6,55	8,73
,1	2,1991	4,40	6,60	8,80
,2	2,2148	4,43	6,64	8,86
,3	2,2305	4,46	6,69	8,92
,4	2,2463	4,49	6,74	8,98
,5	2,2621	4,52	6,79	9,05
,6	2,2780	4,56	6,83	9,11
,7	2,2940	4,59	6,88	9,18
,8	2,3100	4,62	6,93	9 24
,9	2,3261	4,65	6.98	9,30
29,0	2,3422	4,68	7,03	9,37
,1	2,3584	4,72	7,07	9,43
,2	2,3746	4,75	7,12	9,50
,3	2,3910	4,78	7,17	9,56
,4	2,4073	4,81	7,22	9,63
,5	2,4237	4,85	7,27	9,69
,6	2,4401	4,88	7,32	9,76
,7	2,4567	4,91	7,37	9,83
,8	2,4732	4,94	7,42	9,89
,9	2,4899	4 98	7,47	9,96

DEPTH.				
5	6	7	8	9
10,15	12,18	14,21	16,24	18,27
10,23	12,27	14,32	16,36	18,41
10,30	12,36	14,42	16,48	18,54
10,38	12,45	14,53	16,61	18,68
10,45	12,55	14,64	16,73	18,82
10,53	12,64	14,74	16,85	18,95
10,61	12,73	14,85	16,97	19,09
10,68	12,82	14,96	17,10	19,23
10,76	12,91	15,06	17,22	19,37
10,84	13,01	15,18	17,34	19,51
10,92	13,10	15,28	17,46	19,65
11,00	13,19	15,39	17.59	19,79
11,08	13,29	15,50	17,71	19,93
11,15	13,38	15,61	17,84	20,07
11,23	13,48	15,72	17,97	20,22
11,31	13,57	15,83	18,10	20,36
11,39	13,67	15,95	18,22	20,50
11,47	13,76	16,06	18,35	20,65
11,55	13,86	16,17	18,48	20,79
11,63	13,96	16,28	18,61	20,93
11,71	14,05	16,39	18,74	21,08
11,79	14,15	16,51	18,86	21,22
11,87	14,25	16,62	18,99	21,37
11,96	14,35	16,74	19,13	21,52
12,04	14.44	16,85	19.26	21,66
12,12	14,54	16,96	19,39	21,81
12.20	14,64	17,08	19,52	21,96
12,28	14.74	17,19	19,65	22,10
12,37	14,84	17,31	19,78	22,26
12,45	14.94	17,43	19,92	22,41

Diam. in Inches.	DEPTH.			
	1	2	3	4
30,0	2,5066	5,01	7,52	10,03
,1	2,5233	5,05	7,57	10,09
,2	2,5401	5,08	7,62	10,16
,3	2,5570	5,11	7,67	10,23
,4	2,5739	5,15	7,72	10,29
,5	2,5908	5,18	7,77	10,36
,6	2,6078	5,21	7,82	10,43
,7	2,6249	5,25	7,87	10,50
,8	2,6420	5,28	7,93	10,57
,9	2,6592	5,32	7,98	10,64
31,0	2,6764	5,35	8,03	10,71
,1	2,6938	5,39	8,08	10,78
,2	2,7111	5,42	8,13	10,84
,3	2,7285	5,46	8,19	10,91
,4	2,7460	5,49	8,24	10,98
,5	2,7635	5,53	8,29	11,05
,6	2,7811	5,56	8,34	11,12
,7	2,7987	5,60	8,40	11,19
,8	2,8163	5,63	8,45	11,27
,9	2,8341	5,67	8,50	11,34
32,0	2,8519	5,70	8,56	11,41
,1	2,8698	5,74	8,61	11,48
,2	2,8877	5,78	8,66	11,55
,3	2,9057	5,81	8,72	11,62
,4	2,9237	5,85	8,77	11,69
,5	2,9418	5,88	8,83	11,77
,6	2,9599	5,92	8,88	11,84
,7	2,9781	5,96	8,93	11,91
,8	2,9963	5,99	8,99	11,98
,9	3,0148	6,03	9,04	12,06

DEPTH.				
5	6	7	8	9
12,53	15,04	17,54	20,05	22,56
12,62	15.14	17,66	20,18	22,71
12,70	15,24	17,78	20,32	22,86
12,79	15,34	17,90	20,46	23,01
12,87	15,44	18,02	20,59	23,17
12,95	15,55	18,14	20,73	23,32
13,04	15,65	18,25	20,86	23,47
13,12	15,75	18,37	21,00	23,62
13,21	15,85	18,49	21,14	23,78
13,30	15,95	18,61	21,27	23,93
13,38	16,06	18,73	21,41	24,09
13,47	16,16	18,86	21,55	24 25
13,56	16,27	18,98	21,69	24,40
13,64	16,37	19,10	21,83	24,56
13,73	16,48	19,22	21.97	24,71
13,82	16,58	19,34	22,10	24,87
13,91	16,69	19,47	22,25	25,03
13,99	16,79	19,59	22,38	25,18
14,08	16,90	19,71	22,53	25,35
14.17	17,00	19,84	22,67	25,51
14,26	17,11	19,96	22,82	25,67
14,35	17,22	20,09	22,96	25,83
14,44	17,32	20,21	23,10	25,98
14,53	17,43	20,34	23,24	26,15
14,62	17,54	20,46	23,39	26.31
14,71	17,65	20,59	23,53	26,48
14,80	17,76	20,72	23,68	26,64
14,89	17,87	20,85	23,82	26.80
14,98	17,98	20,97	23,97	26,97
15,07	18,09	21,10	24,11	27,13

Diam. in Inches.	DEPTH.			
	1	2	3	4
33,0	3,0330	6,07	9,10	12,13
,1	3,0514	6,10	9,15	12,21
,2	3,0698	6,14	9,21	12,28
,3	3,0884	6,18	9,26	12,35
,4	3,1069	6,21	9,32	12,43
,5	3,1256	6,25	9,38	12,50
,6	3,1443	6,29	9,43	12,58
,7	3,1630	6,33	9,49	12,65
,8	3,1818	6,36	9,55	12,73
,9	3,2007	6,40	9,60	12,80
34,0	3,2196	6,44	9,66	12,88
,1	3,2385	6,48	9,71	12,95
,2	3,2576	6,51	9,77	13,03
,3	3 2765	6,55	9,83	13,11
,4	3,2958	6,59	9,89	13,18
,5	3,3150	6,63	9,95	13,26
,6	3,3342	6,67	10,00	13,34
,7	3,3535	6,71	10,06	13,41
,8	3,3728	6.75	10,12	13,49
,9	3,3923	6,78	10,18	13,57
35,0	3,4117	6,82	10,24	13,69
,1	3,4312	6,86	10.29	13,73
,2	3,4508	6,90	10,35	13,80
,3	3·4705	6,94	10,41	13,[illegible]
,4	3,4902	6,98	10,47	13,96
,5	3,5099	7,02	10,53	14,04
,6	3,5297	7,06	10,59	14,12
,7	3.5496	7,10	10,65	14,20
,8	3,5[illegible]05	7,14	10,71	14,28
,9	3,5894	7,18	10,77	14,36

DEPTH.				
5	6	7	8	9
15,17	18,20	21,23	24,26	27,30
15,26	18,31	21,36	24,41	27,46
15,35	18,42	21,49	24,56	27,63
15,44	18,53	21,62	24,71	27,79
15,53	18,64	21,75	24,85	27,96
15,63	18,75	21,88	25,00	28,13
15,72	18,86	22,01	25,15	28,30
15,82	18,98	22,14	25,30	28,47
15,91	19,09	22,27	25,45	28,63
16,00	19,20	22,40	25,60	28,80
16,10	19,32	22,54	25,75	28,97
16,19	19,43	22,67	25,90	29,14
16,29	19,54	22,80	26,06	29,32
16,38	19,66	22,93	26,21	29,49
16,48	19,77	23,07	25,36	29,66
16,58	19,89	23,21	26,52	29,84
16,67	20,00	23,34	26,67	30,01
16,77	20,12	23,47	26,82	30,18
16,86	20,23	23,61	26,98	30,35
16,96	20,35	23,75	27,14	30,53
17,06	20,47	23,88	27,29	30,70
17,16	20,59	24,01	27,45	30,88
17,25	20,70	24,16	27,61	31,06
17,35	20,82	24,29	27,76	31,24
17,45	20,94	24,43	27,92	31,41
17,55	21,06	24,57	28,08	31,59
17,65	21,18	24,71	28,24	31,76
17,75	21,30	24,85	28,39	31,94
17,85	21,42	24,99	28,55	32,13
17,95	21,53	25,12	28,71	32,30

Diam. in Inches.	DEPTH.			
	1	2	3	4
36.0	3.6065	7.22	10.83	14.44
.1	3.6295	7.26	10.89	14.52
.2	3.6497	7.30	10.95	14.60
.3	3.6699	7.34	11.01	14.68
.4	3.6901	7.38	11.07	14.76
.5	3.7104	7.42	11.13	14.84
.6	3.7308	7.46	11.19	14.92
.7	3.7512	7.50	11.25	15.00
.8	3.7717	7.54	11.32	15.09
.9	3.7922	7.58	11.38	15.17
37.0	3.8128	7.63	11.44	15.25
.1	3.8334	7.67	11.50	15.33
.2	3.8541	7.71	11.56	15.43
.3	3.8749	7.75	11.62	15.50
.4	3.8957	7.79	11.69	15.58
.5	3.9165	7.83	11.75	15.67
.6	3.9374	7.87	11.81	15.75
.7	3.9584	7.92	11.88	15.83
.8	3.9794	7.96	11.94	15.92
.9	4.0005	8.00	12.05	16.00
38.0	4.0217	8.04	12.06	16.09
.1	4.0428	8.09	12.13	16.17
.2	4.0641	8.13	12.17	16.24
.3	4.0854	8.17	12.26	16.34
.4	4.1068	8.21	12.32	16.43
.5	4.1282	8.26	12.38	16.51
.6	4.1496	8.30	12.45	16.60
.7	4.1712	8.34	12.51	16.68
.8	4.1927	8.39	12.58	16.77
.9	4.2144	8.43	12.64	16.86

DEPTH.				
5	6	7	8	9
18.05	21,06	25.27	29.87	32.48
18.15	21,78	25.41	29.03	32.66
18.25	21,90	25.55	29.19	32.84
18.35	22.02	25.69	29.36	33.03
18.45	22.14	25.83	29.52	33.21
18.55	22.26	25.97	29.68	33.39
18.66	22.39	26.11	29.85	33.58
18.76	22.51	26.26	30.01	33.76
18.86	22.63	26.40	30,17	33.94
18.96	22.75	26.55	30.34	34.13
19.07	22.88	26.69	30.51	34.32
19.17	23.00	26.83	30.67	34.50
19.27	23.12	26.98	30.83	34.69
19.37	23.25	27.12	31.00	34.87
19.48	23.37	27.27	31.17	35.06
19.58	23.50	27.41	31.33	35.25
19.69	23.62	27.56	31.50	35.43
19.79	23.75	27.71	31.66	35.62
19.90	23.87	27.86	31.83	35.81
20.00	24.00	28.00	32.00	36.00
20.11	24.13	28.15	32.17	36.19
20.21	24.26	28.30	32.34	36.39
20.30	24.38	28.44	32.49	36.56
20.43	24.51	28.60	32.63	36.77
20.54	24.64	28.75	32.85	36.96
20.64	24.77	28.90	33.02	37.15
20.75	24.90	29.05	33.19	37.34
20.86	25,03	29.20	33.37	37.54
20.96	25.15	29.35	33.54	37.73
21.07	25.29	29.50	33.71	37.93

I

Diam. in Inches.	DEPTH.			
	1	2	3	4
39.0	4.2361	8.47	12.71	16.94
.1	4.2578	8.52	12.77	17.03
.2	4.2796	8.56	12.84	17.12
.3	4.3015	8.60	12.90	17.20
.4	4.3234	8.65	12.97	17.29
.5	4.3454	8.69	13.04	17.38
.6	4.3674	8.73	13.10	17.47
.7	4.3895	8.78	13.17	17.56
.8	4.4117	8.82	13.24	17.65
.9	4.4338	8.87	13.30	17.74
40.0	4.4562	8.91	13.37	17.82
.1	4.4785	8.96	13.43	17.91
.2	4.5008	9.00	13.50	18.00
.3	4.5233	9.05	13.57	18.09
.4	4.5457	9.09	13.64	18.18
.5	4.5683	9.14	13.71	18.27
.6	4.5908	9.18	13.77	18.36
.7	4.6135	9.23	13.84	18.45
.8	4.6362	9.27	13.91	18.54
.9	4.6589	9.32	13.98	18.65
41.0	4.6818	9.36	14.05	18.73
.1	4.7046	9.41	14.11	18.82
.2	4.7275	9.45	14.18	18.91
.3	4.7505	9.50	14.25	19.00
.4	4.7735	9.55	14.32	19.09
.5	4.7966	9.59	14.39	19.19
.6	4.8198	9.64	14.46	19.28
.7	4.8430	9.69	14.53	19.37
.8	4.8662	9.73	14.60	19.46
.9	4.8895	9.78	14.67	19.56

DEPTH.				
5	6	7	8	9
21.18	25.42	29.65	33.89	38.12
21.29	25.55	29.81	34.06	38.32
21.40	25.68	29.95	34.23	38.51
21.51	25.81	30.11	34.41	38.71
21.62	25.94	30.26	34.58	38.91
21.73	26.07	30.42	34.76	39.11
21.84	26.20	30.57	34.94	39.30
21.95	26.33	30.72	35.11	39.50
22.06	26.47	30.89	35.30	39.71
22.17	26.60	31.04	35.47	39.91
22.28	26.74	31.19	35.65	40.11
22.39	26.87	31.35	35.83	40.30
22.50	27.00	31.51	36.01	40.51
22.62	27.14	31.65	36.18	40.71
22.73	27.28	31.82	36.37	40.93
22.84	27.41	31.98	36.54	41.11
22.95	27.55	32.14	36.73	41.32
23.07	27.68	32.29	36.91	41.52
23.18	27.82	32.45	37.09	41.72
23.30	27.95	32.61	37.27	41.93
23.41	28.09	32.77	37.46	42.14
23.52	28.22	32.93	37.03	42.34
23.64	28.36	33.09	37.82	42.54
23.75	28.50	33.25	38.00	42.75
23.87	28.64	33.41	38.19	42.96
23.98	28.78	33.57	38.37	43.17
24.10	28.92	33.74	38.56	43.38
24.22	29.06	33.90	38.74	43.59
24.33	29.19	34.06	38.93	43.80
24.45	29.34	34.23	39.11	41.00

THE

TABLE

CONTINUED,

From 42 to 156 Inches Diameter;

Shewing the

AREA'S of CIRCLES

TO THE

Ten Thouſandth Part of a Gallon, Ale-Meaſure.

D.	.0	.1	.2	.3	.4
42	4.9129	4.9365	4.9598	4.9834	5.0069
43	5.1496	5.1736	5.1977	5.2218	5.2459
44	5.3920	5.4165	5.4411	5.4657	5.4904
45	5.6398	5.6649	5.6901	5.7153	5.7405
46	5.8933	5.9189	5.9446	5.9704	5.9962
47	6.1523	6.1735	6.2048	6.2311	6.2575
48	6.4169	6.4436	6.4705	6.4973	6.5243
49	6.6870	6.7143	6.7417	6.7692	6.7966
50	6.9628	6.9906	7.0186	7.0466	7.0736
51	7.2440	7.2725	7.3010	7.3295	7.3581
52	7.5309	7.5599	7.5890	7.6181	7.6472
53	7.8233	7.8529	7.8825	7.9124	7.9419
54	8.1214	8.1515	8.1816	8.2118	8.2421
55	8.4249	8.4556	8.4863	8.5171	8.5471
56	8.7341	8.7653	8.7966	8.8279	8.8593
57	9.0489	9.0806	9.1124	9.1443	9.1762
58	9.3691	9.4014	9.4338	9.4662	9.4988
59	9.6949	9.7278	9.7608	9.7938	9.8268
60	10.0264	10.0598	10.0933	10.1269	10.1605
61	10.3634	10.3974	10.4314	10.4655	10.4997
62	10.7059	10.7405	10.7751	10.8098	10.8445
63	11.0541	11.0892	11.1254	11.1596	11.1949
64	11.4078	11.4434	11.4792	11.5150	11.5598
65	11.7670	11.8033	11.8396	11.8759	11.9123
66	12.1319	12.1687	12.2055	12.2424	12.2794
67	12.5023	12.5357	12.5771	12.6145	12.6520
68	12.8723	12.9162	12.9542	12.9922	13.0303
69	13.2599	13.2982	13.3368	13.3754	13.4140
70	13.6470	13.6860	13.7251	13.7645	13.8034
71	14.0397	14.0793	14.1189	14.1586	14.1983

.5	.6	.7	.8	.9
5.0306	5.0543	5.0780	5.1019	5.1257
5.2701	5.2944	5.3187	5.3430	5.3676
5.5152	5.5400	5.6649	5.5898	5.6148
5.7659	5.7912	5.8567	5.8421	5.8677
6.0221	6.0480	6.0740	6.1000	6.1261
6.2839	6.3104	6.3369	6.3635	6.3902
6.5513	6.5783	6.6054	6.6325	6.6600
6.8242	6.8518	6.8794	6.9072	6.9349
7.1027	7.1309	7.1591	7.1873	7.2157
7.3868	7.4155	7.4443	7.4731	7.5020
7.6764	7.7057	7.7350	7.7644	7.7939
7.9717	8.0015	8.0314	8.0613	8.0913
8.2724	8.3028	8.3333	8.3638	8.3943
8.5788	8.6097	8.6407	8.6768	8.7029
8.8907	8.9222	8.9538	8.9854	~~8.9191~~ 9.0171
9.2082	9.2403	9.2734	9.3046	9.3368
9.5313	9.5639	9.5966	9.6293	9.6621
9.8600	9.8931	9.9263	9.9596	9.9930
10.1942	10.2279	10.2617	10.2955	10.3294
10.5339	10.5682	10.6026	10.6370	10.6714
10.8793	10.9141	10.9490	10.9840	11.0190
11.2302	11.2656	11.3011	11.3366	11.3721
11.5867	11.6227	11.6587	11.6947	11.7309
11.9488	11.9853	12.0419	12.0585	12.0952
12.3164	12.3535	12.3906	12.4278	12.4650
12.6896	12.7272	12.7649	12.8027	12.8405
13.0684	13.1066	13.1448	13.1831	13.2215
13.4527	13.4915	13.5303	13.5691	13.6080
13.8426	13.8819	13.9212	13.9607	14.0002
14.2381	14.2780	14.3179	14.3579	14.3979

Dia	.0	1.	.2	.3	.4
72	14.4380	14.4781	14.5183	14.5585	14.5988
73	14.8418	14.8825	14.9232	14.9640	15.0049
74	15.2512	15.2925	15.3338	15.3751	15.4165
75	15.6662	15.7080	15.7499	15.7918	15.8337
76	16.0867	16.1291	16.1715	16.2140	16.2565
77	16.5129	16.5558	16.5988	16.6418	16.6849
78	16.9445	16.9880	17.0316	17.0751	17.1188
79	17.3818	17.4258	17.4699	17.5141	17.5583
80	17.8246	17.8692	17.9139	17.9586	18.0033
81	18.2730	18.3182	18.3634	18.4086	18.4540
82	18.7270	18.7727	18.8185	18.8643	18.9102
83	19.1866	19.2328	19.2791	19.3255	19.3719
84	19.6517	19.6985	19.7454	19.7923	19.8393
85	20.1223	20.1697	20.2172	20.2646	20.3122
86	20.5980	20.6465	20.6945	20.7246	20.7907
87	21.0804	21.1289	21.1775	21.2261	21.2747
88	21.5678	21.6169	21.6660	21.7151	21.7643
89	22.0608	22.1104	22.1600	22.2098	22.2595
90	22.5593	22.6095	22.6597	22.7100	22.7603
91	23.0634	23.1141	23.1649	23.2157	23.2666
92	23.5731	23.6244	23.6757	23.7271	23.7785
93	24.0883	24.1402	24.1920	24.2450	24.2960
94	24.6091	24.6615	24.7140	24.7665	24.8190
95	25.1355	25.1885	25.2415	25.2945	25.2376
96	25.6675	25.7210	25.7745	25.8282	25.8818
97	26.2050	26.2591	26.3132	26.3673	26.4216
98	26.7481	26.8027	26.8574	26.9121	26.9669
99	27.2968	27.3591	27.4072	27.4625	27.5178
100	27.8510	27.9067	27.9625	28.0184	28.9743
101	28.4108	28.4671	28.5234	28.5798	28.6363

.5	.6	.7	.8	.9
14.6392	14.6796	14.7201	14.7606	14.8012
15.0458	15.0868	15.1278	15.1689	15.2100
15.4580	15.4950	15.5411	15.5827	15.6244
15.8758	15.9178	15.9600	16.0022	16.0444
16.2991	16.3417	16.3844	16 4272	16.4700
16.7280	16.7712	16.8145	16.8578	16.9011
17.1625	17.2062	17.2500	17.2939	17.3378
17.6025	17.6468	17.6912	17.7356	17.7801
18.0481	18.0930	18.1379	18 1829	11.2280
18.4993	18.5448	18.5902	18.6358	18.6814
18.9561	19.0021	19.0481	19.0947	19.1403
19.4184	19.4650	19.5115	19.5582	19.6049
19.8863	19.9334	19.9806	20.0278	20.0750
20.3598	20.4074	20.4551	20.5029	20.5507
20.8388	20.8870	20.9353	20.9836	21.0320
21.3234	21.3722	21.4210	21.4699	21.5188
21.8136	21.8629	21.9123	21.9617	22.0112
22.3093	22.3592	22.4092	22.4592	22.5092
22.8107	22.8611	22.9116	22.9621	23.0128
23.3176	23.3685	23.4196	23.4707	23.5220
23.8300	23.8816	23.9332	23.9848	24.0367
24.3480	24.4001	24.4523	24.5045	24.5568
24.8716	24.9243	24.9770	25.0298	25.0826
25.4008	25.4540	25.5073	25.5606	25.6140
25.9355	25.9893	26.0432	26.0671	26.1510
26.4759	26.5302	26.5846	26.6090	26.6935
27.0127	27.0766	27.1316	27.1866	27.2416
27.5732	27.6286	27.6841	27.7397	27.7953
28.1302	28.1862	28.2423	28.2984	28.3546
28.6928	28.7494	28.8069	28.8627	28.9194

Dia	.0	.1	2.	.3	.4
102	28.9762	29.0330	29.0899	29.1469	29.2039
103	29.5471	29.6045	29.6620	29.2195	29.7771
104	30.1236	30.1816	30.2396	30.2977	30.3558
105	30.7057	30.7642	30.8228	30.8814	30.9401
106	31.2934	31.3525	31.4116	31.4708	31.5300
107	31.8866	31.9462	32.0059	32.0657	32.1255
108	32.4854	32.5456	32.6058	32.6661	32.7265
109	33.0898	33.1505	33 2113	33.2702	33.3331
110	33.6997	33.7610	33.8224	33.8838	33.9452
111	34.3152	34.3771	34.4399	34.5010	34.5630
112	34.9363	34 9987	35.0612	35.1237	35.1863
113	35.5629	35.6259	35.6889	35.7520	35.8152
114	36.1952	36.2587	36.3223	36.3859	36.4496
115	36.8309	36.8970	36.9612	37.0254	37.0896
116	37.4762	37.5409	37.6056	37.6704	37.7352
117	38.1252	38.1904	38.2557	38.3210	38.3864
118	38.7797	38.8455	38.9113	38.9772	39.0431
119	39.4398	39.5061	39.5725	39 6389	39.7054
120	40.1054	40.1725	40.2392	40.3062	40 3733
121	40.7766	40.8441	40.9116	40.9791	41.0467
122	41.4534	41.5214	41.5895	41.6575	41.7257
123	42.1358	42.2043	42 2729	42.3416	42.4103
124	42.8237	42 8928	42 9619	43.0312	43.1004
125	43.5172	43.5868	43.6566	43.7263	43.7961
126	44.2162	44.2862	44.3567	44.4274	44.4974
127	44.9209	44.1967	45.0625	45.1331	45.2043
128	45.6311	45.7024	45.7738	45.8452	45.9167
129	46.3468	46.4187	46.4907	46.5627	46.6347
130	47.0682	47.1406	47.2131	47.2857	47.3583
131	47.7951	47.8681	47.9412	48.0143	48.0874

	·5	·6	·7	·8	·9
	29.2610	29.3181	29.3743	29.4325	29.4898
	26.8347	29.8924	29.9501	30 0079	30.0657
	30.4140	30.4722	30.5305	30.5889	30.6473
	30.9989	31.0577	31.1165	31.1754	31.2344
	31.5892	31.6487	31.7081	31.7675	31,8270
	32.1853	32.2452	32.3052	32.3652	32.4252
	32.7869	32 8474	32.9079	32.9685	33.0291
	33.3940	33.4551	33.5161	33.5773	33.6385
	34.0068	34.0683	34.1300	34.1917	34.2534
	34.6251	34.6872	34.7494	34.8116	34.8739
	35.2489	35.3116	35.3744	35.4372	35.5000
	35.8784	35.9416	36.0049	36.0683	36.1317
	36.5134	36.5772	36.6410	36.7049	36.7689
	37.1539	37.2183	37.2827	37.3472	37.4117
	37.8001	37.8650	37.9300	37.9950	38 0601
	38.4518	38.5173	38.5828	38.6484	38.7140
	39.1091	39.1751	39.2412	39 3073	39.3735
	39.7719	39.8385	39 9052	39 9719	40.0386
	40.4403	40.5075	40.5747	40.6420	40.7093
	41.1143	41.1820	41.2498	41.3176	41.3855
	41.7939	41.8622	41.9305	41.9989	42.0673
	42.4790	42.5479	42.6167	42.6858	42.7547
	43.1697	43 2391	43.3086	43.3780	43.4476
	43.8660	43.9360	44.0059	44.0760	44.1461
	44.5679	44.6384	44.7089	44.7795	44.8502
	45.2753	45.3463	45.4174	45.4886	45 5598
	45.9883	46.0599	46.1312	46.2032	46.2750
	46.7068	46.7790	46.8512	46.9235	46.9958
	47.4309	47.5037	47.5764	47 6493	47.7222
	48.1606	48.2339	48 3073	48.3806	48.4541

Dia	.0	.1	.2	.3	.4
132	48,5276	48,6011	48,6747	48,7484	48,8221
133	49,2656	49,3397	49,4139	49,4881	49,5624
134	50,0093	50,0839	50,1586	50,2334	50,3083
135	50,7584	50,8340	50,9099	50,9842	51,0600
136	51,5132	51,5890	51,6650	51,7410	51,8170
137	52,2740	52,3500	52,4262	52,5030	52,5729
138	53,0394	53,1164	53,1933	53,2703	53,3474
139	53,8109	53,8884	53,9659	54,0434	54,1211
140	54,5880	54,6660	54,7440	54,8222	54,9003
141	55,3706	55,4491	55,5268	55,6064	55,6852
142	56,1587	56,2379	56,3170	56,3963	56,4756
143	56,9525	57,0322	57,1119	57,1917	57,2716
144	57,7518	57,8321	57,9124	57,9927	58,0731
145	58,5567	58,6375	58,7184	58,7993	58,8802
146	59,3672	59,4485	59,5299	59,6114	59,6929
147	60,1832	60,2651	60,3471	60,4291	60,5112
148	61,0048	61,0873	61,1698	61,2524	61,3350
149	61,8320	61,9150	61,9981	62,0812	62,1644
150	62,6648	62,7483	62,8320	62,9257	62,9994
151	63,5030	63,5872	63,6714	63,7556	63,8400
152	64,3470	64,4316	64,5164	64,6012	64,6861
153	65,1964	65,2817	65,3670	65,4523	65,5377
154	66,0514	66,1372	66,2231	66,3090	66,3950
155	66,9120	66,9984	67,8448	67,1713	67,2578
156					

.5	.6	.7	.8	.9
48,8959	48,9697	48,0436	49,1176	49,1916
49,6367	49,7111	49,7856	49,8601	49,9346
50,3832	50,4581	50,5331	50,6082	50,6833
51,1351	51,2110	51,2861	51,3620	51,4374
51,8930	51,9690	52,0450	52,1210	52,1972
52,6560	52,7324	52,8090	52,8860	52,9630
53,4245	53,5017	53.5789	53,6552	53,7335
54,1987	54,2765	54,3543	54,4321	54,5100
54,9786	55,0569	55,1325	55,2136	55,2921
55,7640	55,8428	55,9217	56,0007	56,0797
56,5549	56,6343	56,7138	56,7933	56,8729
57,3515	57,4314	57,5114	57,5915	57,6716
58,1536	58,2341	58,3147	58,3953	58,4760
58,9613	59,0423	59,1235	59,2047	59,2859
59,7745	59,8561	59,9378	60.0196	60,1014
60,5933	60,6755	60,7578	60,8401	60,9224
61,4177	61,5005	61,5833	61,6661	61,7490
62,2477	62,3310	62,4143	62,4978	62,5812
63,0832	63,1671	63,2510	63,3350	63,4190
63,9243	64,0087	64,0932	64,1777	64,2623
64,7710	64,8560	64,9410	65,0261	65,1112
66,6232	65,7088	65,7943	65,8800	65,9657
66,4810	66,5671	66,6533	66,7395	66,8257
67,3444	67,4311	67,5178	67,6045	67,6013

D.	.0	.1	.2	.3	.4
1	0,0034	0,0041	0,0049	0,0057	0,0067
2	0,0136	0,0150	0,0165	0,0180	0,0196
3	0,0306	0,0327	0,0348	0,0370	0,0393
4	0,0544	0,0572	0,0600	0,0629	0,0658
5	0,0850	0,0884	0,0919	0,0955	0,0991
6	0,1224	0,1265	0,1307	0,1349	0,1393
7	0,1666	0,1714	0,1763	0,1812	0,1862
8	0,2176	0,2231	0,2286	0,2342	0,2399
9	0,2754	0,2816	0,2878	0,2941	0,3004
10	0,3400	0,3468	0,3538	0,3607	0,3677
11	0,4114	0,4189	0,4265	0,4341	0,4419
12	0,4896	0,4978	0,5060	0,5144	0,5228
13	0,5746	0,5835	0,5924	0,6014	0,6105
14	0,6664	0,6760	0,6856	0,6953	0,7050
15	0,7650	0,7752	0,7855	0,7959	0,8063
16	0,8704	0,8813	0,8923	0,9033	0,9145
17	0,9826	0,9943	1,0059	1,0176	1,0294
18	1,1016	1,1139	1,1262	1,1386	1,1511
19	1,2274	1,2404	1,2534	1,2665	1,2796
20	1,3600	1,3736	1,3873	1,4011	1,4149
21	1,4994	1,5137	1,5281	1,5425	1,5571
22	1,6456	1,6606	1,6757	1,6908	1,7060
23	1,7986	1,8143	1,8300	1,8458	1,8617
24	1,9584	1,9748	1,9912	2,0077	2,0242
25	2,1250	2,1420	2,1591	2,1763	2,1935
26	2,2984	2,3161	2,3339	2,3517	2,3697
27	2,4786	2,4970	2,5155	2,5330	2,5526
28	2,6656	2,6848	2,7028	2,7230	2,7423
29	2,8594	2,8792	2,8990	2,9189	2,9388
30	3,0600	3,0804	3,1009	3,1215	3,1421

·5	.6	·7	.8	·9
0,0077	0,0087	0,0098	0,0110	0,0123
0,0213	0,0230	0,0248	0,0267	0,0286
0,0417	0,0441	0,0465	0,0491	0,0514
0,0689	0,0719	0,0751	0,0781	0,0816
0,1029	0,1066	0,1105	0,1144	0,1184
0,1437	0,1481	0,1526	0,1572	0,1619
0,1913	0,1964	0,2016	0,2069	0,2122
0,2457	0,2515	0,2573	0,2633	0,2693
0,3069	0,3133	0,3199	0,3265	0,3332
0,3748	0,3820	0.3893	0,3966	0,4040
0,4496	0,4575	0,4654	0,4734	0,4815
0,5312	0,5398	0,5484	0,5571	0,5658
0,6196	0,6289	0,6381	0,6475	0,6569
0,7148	0.7247	0,7347	0,7447	0,7548
0,8168	0,8274	0,8381	0,8488	0,8596
0,9256	0,9369	0,9442	0,9596	0,9711
1,0412	1,0532	1,0652	1,0773	1,0894
1,1636	1,1736	1,1889	1,2017	1,2145
1,2928	1,3061	1,3195	1,3329	1,3464
1,4288	1.4428	1,4569	1,4710	1,4852
1,5716	1,5863	1,6010	1,6158	1,6307
1,7212	1,7366	1,7520	1,7675	1,7830
1,8776	1,8937	1,9697	1,9259	1,9421
2,0408	2,0575	2,0743	2,0911	2,1080
2,2108	2,2282	2,2457	2,2632	2,2808
2,3876	2,4057	2,4238	2,4420	2,4663
2,5712	2,5900	2,6088	2,6277	2,6466
2,7616	2,7811	2,8005	2,8201	2,8396
2,9588	2,9789	2,9991	3,0193	3,0396
3.1628	3,1836	3,2045	3,2254	3,2464

D.	.0	.1	.2	.3	.4
31	3,2674	3,2885	3,3097	3,3309	3,3523
32	3,4816	3,5034	3,5253	3,5472	3,5692
33	3,7026	3,7251	3,7476	3,7702	3,7929
34	3,9304	3,9536	3,9768	4,0001	4,0234
35	4,1650	4,1888	4,2127	4,2367	4,2607
36	4,4064	4,4309	4,4555	4,4801	4,5049
37	4,6546	4,6798	4,7051	4,7304	4,7558
38	4,9096	4,9355	4,9614	4,9874	5,0135
39	5,1714	5,1980	5,2246	5,2513	5,2780
40	5,4400	5,4672	5,4945	5,5219	5,5493
41	5,7154	5,7433	5,7713	5,7993	5,8275
42	5,9976	6,0262	6,0549	6,0836	6,1124
43	6,2866	6,3159	6,3452	6,3746	6,4041
44	6,5824	6,6142	6,6424	6,6725	6,7026
45	6,8850	6,9156	6,9463	6,9771	7,0079
46	7,1944	7,2257	7,2571	7,2885	7,3201
47	7,5106	7,5426	7,5747	7,6068	7,6390
48	7,8336	7,8663	7,8990	7,9318	7,9647
49	8,1634	8,1968	8,2302	8,2637	8,2972
50	8,5000	8,5340	8,5681	8,6024	8,6365
51	8,8434	8,8781	8,9129	8,9477	8,9827
52	9,1936	9,2290	9,2645	9,3000	9,3356
53	9,5506	9,5867	9,6228	9,6590	9,6953
54	9,9144	9,9512	9,6880	10,0249	10,0618
55	10,2850	10,3224	10,3599	10,3975	10,4351
56	10,6624	10,7005	10,7387	10,7765	10,8153
57	11,0466	11,0854	11,1243	11,1632	11,2022
58	11,4370	11,4771	11,5166	11,5562	11,5959
59	11,8354	11,8756	11,9158	11,9561	11,9964
60	12,2400	12,2808	12,3217	12,3627	12,4037

	·5	.6	·7	.8	·9
—	3,3736	3,3951	3,4166	3,4382	3,4599
	3,5912	3,6134	3,6356	3,6579	3,6802
	3,8156	3,8385	3,8613	3,8843	3,9073
	4,0468	4,0703	4,0939	4,1175	4,1412
	4,2848	4,3090	4,3333	4,3576	4,3820
	4,5296	4,5545	4,5794	4,6044	4,6295
	4,7812	4,8068	4,8324	4,8581	4,8838
	5,0396	5,0659	5,0921	5,1185	5,1449
	5,2048	5.3317	5,3587	5,3857	5,4128
	4,5768	5,6044	5,6321	5,6598	5,6876
—	5,8556	5,8839	5,9122	5,9406	5,9691
	6,1412	6,1702	6,1992	6,2283	6,2574
	6,4336	6,4633	6,4929	6,5227	6,5525
	6,7328	6,7631	6.7935	6,8239	6,8544
	7,0388	7,0698	7,1009	7,1320	7,1632
	7,3516	7,3833	7,4150	7,4468	7,4787
	7,6712	7,7036	7,7360	7,7685	7,8010
	7,9976	8,0307	8,0637	8,0968	8,1301
	8,3308	8,3645	8,3983	8,4321	8,4660
	8,6708	8,7052	8,7397	8,7742	8,8088
—	9,0176	9,0527	9,0878	9,1230	9,1583
	9,3712	9,4070	9,4428	9,4787	9,5146
	9,7316	9,7681	9,8045	9,8411	9,8577
	10,0988	10,1359	10,1731	10,2103	10,2476
	10,4728	10,5106	10,5485	10,5864	10,6244
	10,8536	10,8921	10,9306	10,9692	11,0079
	11,2412	11,2804	11,3196	11,3589	11,3982
	11,6356	11,6755	11,7153	11,7553	11,7953
	12,0368	12,0773	12,1179	12,1585	12,1992
	12,4448	12,4860	12,5273	11,5686	12,6100

THE foregoing Table is uſeful in Gauging of Wine-Veſſels.

Example. There is a Brandy Butt ſuppoſed to be in the Form of the middle Fruſtum of a Spheroid: The Bung-Diameter is 32 Inches, the Head-Diameter 24 Inches, and the Length 40 Inches. What is the Content of this Caſk in Wine-Gallons?

From the Bung Diameter ———————— 32
Subtract the Head-Diameter ———————— 24

The Difference is ———————— 8

Find 8 in the firſt Colum of this Table, Page 126, againſt it is 5.6, which added to 24, the Head-Diameter is 29.6 the Mean Diameter of this Caſk. Now againſt 29 in this Table, and under .6 is 2.9789 the Area. Which multiplied by 40, the Length of the Caſk, gives 119.156, the Content ſought. The like for any other Caſk whoſe Mean Diameter is known.

TABLES

OF

EXCISE,

According to the

Reſpective Allowances made to COMMON BREWERS, both in LONDON and the COUNTRY.

Exciſe for Common Brewers in the Country at 5*s.* *per* Barrel, the Allowance being 2½ Barrels in every 23.

	0					¼					½					¾				
B.	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	4	5	1	21	0	5	6	3	9	0	6	8	0	20	0	7	9	2	8
2	0	8	10	3	19	0	10	0	1	7	0	11	1	2	18	0	12	3	0	6
3	0	13	4	1	17	0	14	5	3	5	0	15	7	0	16	0	16	8	2	4
4	0	17	9	3	15	0	18	11	1	3	1	0	0	2	14	1	1	2	0	2
5	1	2	3	1	13	1	3	4	3	1	1	4	6	0	12	1	5	7	2	0
6	1	6	8	3	11	1	7	10	0	22	1	8	11	2	10	1	10	0	3	21
7	1	11	2	1	9	1	12	3	2	20	1	13	5	0	8	1	14	6	1	19
8	1	15	7	3	7	1	16	9	0	18	1	17	10	2	6	1	18	11	3	17
9	2	0	1	1	5	2	1	2	2	16	2	2	4	0	4	2	3	5	1	15
10	2	4	6	2	3	2	5	8	0	14	2	6	9	2	2	2	7	10	3	13
11	2	9	0	1	1	2	10	1	2	12	2	11	3	0	0	2	12	4	1	11
12	2	13	5	2	22	2	14	7	0	10	2	15	8	1	21	2	16	9	3	9
13	2	17	11	0	21	2	19	0	2	8	3	0	1	3	19	3	1	3	1	7
14	3	2	4	2	18	3	3	6	0	6	3	4	7	1	17	3	5	8	3	5
15	3	6	10	0	16	3	7	11	2	4	3	9	0	3	15	3	10	2	1	3
16	3	11	3	2	14	3	12	5	0	2	3	11	6	1	13	3	14	7	3	1
17	3	15	9	0	12	3	16	10	3	0	3	17	11	3	11	3	19	1	0	22
18	4	0	2	2	10	4	1	3	1	21	4	2	5	1	9	4	3	6	2	20
19	4	4	8	0	8	4	5	9	3	19	4	6	10	3	7	4	8	0	0	18
20	4	9	1	2	6	4	10	2	1	17	4	11	4	1	5	4	12	5	2	16
21	4	13	7	0	4	4	14	8	3	15	4	15	9	3	3	4	16	11	0	14
22	4	18	0	2	2	5	19	1	1	13	4	0	3	1	1	5	1	4	2	12

Excise for Common Brewers in the Country at 1 *s.* 4 *d.* *per* Barrel, *&c.*

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f.*
23	5	2	6	0	713	158	17	6	0
46	10	5	0	0	736	164	0	0	0
69	15	7	6	0	759	169	2	6	0
92	20	10	0	0	782	174	5	0	0
115	25	12	6	0	805	179	7	6	0
138	30	15	0	0	828	184	10	0	0
161	35	17	6	0	851	189	12	6	0
184	41	0	0	0	874	194	15	0	0
207	46	2	6	0	897	199	17	6	0
230	51	5	0	0	920	205	0	0	0
253	56	7	6	0	934	210	2	6	0
276	61	10	0	0	966	215	5	0	0
299	66	12	6	0	989	220	7	6	0
322	71	15	0	0	1012	225	10	0	0
345	76	17	6	0	1035	230	12	6	0
368	82	0	0	0	1058	235	15	0	0
391	87	2	6	0	1081	240	17	6	0
414	92	5	0	0	1104	246	0	0	0
437	97	7	6	0	1127	251	2	6	0
460	102	10	0	0	1150	256	5	0	0
483	107	12	6	0	1173	261	7	6	0
506	112	15	0	0	1196	266	10	0	0
529	117	17	6	0	1219	271	12	6	0
552	123	0	0	0	1242	276	15	0	0
575	128	2	6	0	1265	281	17	6	0
598	133	5	0	0	1288	287	0	0	0
621	138	7	6	0	1311	292	2	6	0
644	143	10	0	0	1334	297	5	0	0
667	148	12	6	0	1357	302	7	6	0
690	153	15	0	0	1380	307	10	0	0

Exciſe for Common Brewers in the Country at 5 *s. per* Barrel, *&c.*

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	312	12	6	0	2093	466	7	6	0
1426	317	15	0	0	2116	471	10	0	0
1449	322	17	6	0	2139	476	12	6	0
1472	328	0	0	0	2162	481	15	0	0
1495	333	2	6	0	2185	486	17	6	0
1518	338	5	0	0	2208	492	0	0	0
1541	343	7	6	0	2231	497	2	6	0
1564	348	10	0	0	2254	502	5	0	0
1587	353	12	6	0	2277	507	7	6	0
1610	358	15	0	0	2300	512	10	0	0
1633	363	17	6	0	2323	517	12	6	0
1656	369	0	0	0	2346	522	15	0	0
1679	374	2	6	0	2369	527	17	6	0
1702	379	5	0	0	2392	533	0	0	0
1725	384	7	6	0	2415	538	2	6	0
1748	389	10	0	0	2438	543	5	0	0
1771	394	12	6	0	2461	548	7	6	0
1794	399	15	0	0	2484	553	10	0	0
1817	404	17	6	0	2507	558	12	6	0
1840	410	0	0	0	2530	563	15	0	0
1863	415	2	6	0	5060	1127	10	0	0
1886	420	5	0	0	7590	1691	5	0	0
1909	425	7	6	0	10120	2255	0	0	0
1932	430	10	0	0	12650	2818	15	0	0
1955	435	12	6	0	15180	3282	10	0	0
1978	440	15	0	0	17710	3946	5	0	0
2001	445	17	6	0	20240	4510	0	0	0
2024	451	0	0	0	22770	5073	15	0	0
2047	456	2	6	0	25300	5637	10	0	0
2070	461	5	0	0	27830	6201	5	0	0

Exciſe for Common Brewers in the Country, at 1 *s.* 4 *d.* *per* Barrel, the Allowances being 2½ Barrels in every 23.

	0					¼					½					¾				
B.	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	1	2	1	1	0	1	5	3	7	0	1	9	1	13	0	2	0	3	19
2	0	2	4	2	2	0	2	8	0	8	0	2	11	2	14	0	3	3	0	20
3	0	3	6	3	3	0	3	10	1	9	0	4	1	3	15	0	4	5	1	21
4	0	4	9	0	4	0	5	0	2	10	0	5	4	0	16	0	5	7	2	22
5	0	5	11	1	5	0	6	2	3	11	0	6	6	1	17	0	6	10	0	0
6	0	7	1	2	6	0	7	5	0	12	0	7	8	2	18	0	8	0	1	1
7	0	8	3	3	7	0	8	7	1	13	0	8	10	3	19	0	9	2	2	2
8	0	9	6	0	8	0	9	9	2	14	0	10	1	0	20	0	10	4	3	3
9	0	10	8	1	9	0	10	11	3	15	0	11	3	1	21	0	11	7	0	4
10	0	11	10	2	10	0	12	2	0	16	0	12	5	2	22	0	12	9	1	5
11	0	13	0	3	11	0	13	4	1	17	0	13	8	0	0	0	13	11	2	6
12	0	14	3	0	12	0	14	6	2	18	0	14	10	1	1	0	15	1	3	7
13	0	15	5	1	13	0	15	8	3	19	0	16	0	2	2	0	16	4	0	8
14	0	16	7	2	14	0	16	11	0	20	0	17	2	3	3	0	17	6	1	9
15	0	17	9	3	15	0	18	1	1	21	0	18	5	0	4	0	18	8	2	10
16	0	19	0	0	16	0	19	3	2	22	0	19	7	1	5	0	19	10	3	11
17	1	0	2	1	17	1	0	6	0	0	1	0	9	2	6	1	1	1	0	12
18	1	1	4	2	18	1	1	8	1	1	1	1	11	3	7	1	2	3	1	13
19	1	2	6	3	19	1	2	10	2	2	1	3	2	0	8	1	3	5	2	14
20	1	3	9	0	20	1	4	0	3	3	1	4	4	1	9	1	4	7	3	15
21	1	4	11	1	21	1	5	3	0	4	1	5	6	2	10	1	5	10	0	16
22	1	6	1	2	22	1	6	5	1	1	1	6	8	3	11	1	7	0	1	17

Excise for Common Brewers in the Country, at 5 *s. per* Barrel, &c.

Bar.	*l.*	*s.*	*d.*	*f.*	Bar.	*l.*	*s.*	*d.*	*f.*
23	1	7	4	0	713	42	7	4	0
46	2	4	8	0	736	43	14	8	0
69	4	2	0	0	759	45	2	0	0
92	5	9	4	0	782	46	9	4	0
115	6	16	8	0	805	47	16	8	0
138	8	4	0	0	828	49	4	0	0
161	9	11	4	0	851	50	11	4	0
184	10	18	8	0	874	51	18	8	0
207	12	6	0	0	897	53	6	0	0
230	13	13	4	0	920	54	13	4	0
253	15	0	8	0	943	56	0	8	0
276	16	8	0	0	966	57	8	0	0
299	17	15	4	0	989	58	15	4	0
322	19	2	8	0	1012	60	2	8	0
345	20	10	0	0	1035	61	10	0	0
368	21	17	4	0	1058	62	17	4	0
391	23	4	8	0	1081	64	4	8	0
414	24	12	0	0	1104	65	12	0	0
437	25	19	4	0	1127	66	19	4	0
460	27	6	8	0	1150	68	6	8	0
483	28	14	0	0	1173	69	14	0	0
506	30	1	4	0	1196	71	1	4	0
529	31	8	8	0	1219	72	8	8	0
552	32	16	0	0	1242	73	16	0	0
575	34	3	4	0	1265	75	3	4	0
598	35	10	8	0	1288	76	10	8	0
621	36	18	0	0	1311	77	18	0	0
644	38	5	4	0	1334	79	5	4	0
667	39	12	8	0	1357	80	12	8	0
690	41	0	0	0	1380	82	0	0	0

Exciſe for Common Brewers in the Country at 1 s. 4 d. per Barrel, &c.

Bar.	l.	s.	d.	f.	Bar.	l.	s.	d.	f.
1403	83	7	4	0	2093	124	7	4	0
1426	84	14	8	0	2116	125	14	8	0
1449	85	2	0	0	2139	127	2	0	0
1472	87	9	4	0	2162	128	9	4	0
1495	88	16	8	0	2185	129	16	8	0
1518	90	4	0	0	2208	131	4	0	0
1541	91	11	4	0	2231	132	11	4	0
1564	92	18	8	0	2254	133	18	8	0
1587	94	6	0	0	2277	135	6	0	0
1610	95	13	4	0	2300	136	13	4	0
1633	97	0	8	0	2323	138	0	8	0
1656	98	8	0	0	2346	139	8	0	0
1679	99	15	4	0	2369	140	15	4	0
1702	101	2	8	0	2392	142	2	8	0
1725	102	10	0	0	2415	143	10	0	0
1748	103	17	4	0	2438	144	17	4	0
1771	105	4	8	0	2461	146	4	8	0
1794	106	12	0	0	2484	147	12	0	0
1817	107	19	4	0	2507	148	19	4	0
1840	109	6	8	0	2530	150	6	8	0
1863	110	14	0	0	5060	300	13	4	0
1836	112	1	4	0	7590	451	0	0	0
1909	113	8	8	0	10120	601	6	8	0
1932	114	16	0	0	12650	751	13	4	0
1955	116	3	4	0	15180	902	0	0	0
1978	117	10	8	0	17710	1052	6	8	0
2001	118	18	0	0	20240	1202	13	4	0
2024	120	5	4	0	22770	1353	0	0	0
2047	121	12	8	0	25300	1503	6	8	0
2070	123	0	0	0	27830	1653	13	4	0

Exciſe for Common Brewers in *London*, at 5 *s.* *per* Barrel, the Allowances being 3 Barrels in every 23 of Beer.

	0					¼					½					¾				
B.	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	4	4	0	16	0	5	5	0	20	0	6	6	1	1	0	7	7	1	5
2	0	8	8	1	9	0	9	9	1	13	0	10	10	1	17	0	11	11	1	21
3	0	13	0	2	2	0	14	1	2	6	0	15	2	2	10	0	16	3	2	14
4	0	17	4	2	18	0	18	5	2	22	0	19	6	3	3	1	0	7	3	7
5	1	1	8	3	11	1	2	9	3	15	1	3	10	3	19	1	5	0	0	0
6	1	6	1	0	4	1	7	2	0	8	1	8	3	0	12	1	9	4	0	16
7	1	10	5	0	20	1	11	6	1	1	1	12	7	1	5	1	13	8	1	9
8	1	14	9	1	13	1	15	10	1	17	1	16	11	1	21	1	18	0	2	2
9	1	19	1	2	6	2	0	2	2	10	2	1	3	2	14	2	2	4	2	18
10	2	3	5	2	22	2	4	6	3	3	2	5	7	3	7	2	6	8	3	11
11	2	7	9	3	15	2	8	10	3	19	2	10	0	0	0	2	11	1	0	4
12	2	12	2	0	8	2	13	3	0	12	2	14	4	0	16	2	15	5	0	20
13	2	16	6	1	1	2	17	7	1	5	2	18	8	1	9	2	19	9	1	13
14	3	0	10	1	17	3	1	11	1	21	3	3	0	2	2	3	4	1	2	6
15	3	5	2	2	10	3	6	3	2	14	3	7	4	2	18	3	8	5	3	22
16	3	9	6	3	3	3	10	7	3	7	3	11	8	3	11	3	12	9	3	15
17	3	13	10	3	19	3	15	0	0	0	3	16	1	0	4	3	17	2	0	8
18	3	18	3	0	12	3	19	4	0	16	4	0	5	0	20	4	1	6	1	1
19	4	2	7	1	5	4	3	8	1	9	4	4	9	1	13	4	5	10	1	17
20	4	6	11	1	21	4	8	0	2	2	4	9	1	2	6	4	10	2	2	10
21	4	11	3	2	14	4	12	4	2	18	4	13	5	2	22	4	14	6	3	3
22	4	15	7	3	7	4	16	8	3	11	4	17	9	3	15	4	18	10	3	19

Exciſe for Common Brew[illegible] *London*, at 5 *s. per* Barrel, [illegible]

Bar.	*l.*	Bar.	*l.*	[illegible]	*l.*	Bar.	*l.*
23	5	713	155	1403	305	2093	455
46	10	736	160	1426	310	2116	460
69	15	759	165	1449	315	2139	465
92	20	782	170	1472	320	2162	470
115	25	805	175	1495	325	2185	475
138	30	828	180	1518	330	2208	480
161	35	851	185	1541	335	2231	485
184	40	874	190	1564	340	2254	490
207	45	897	195	1587	345	2277	495
230	50	920	200	1610	350	2300	500
253	55	943	205	1633	355	2323	505
276	60	966	210	1656	360	2346	510
299	65	989	215	1679	365	2369	515
322	70	1012	220	1702	370	2392	520
345	75	1035	225	1725	375	2415	525
368	80	1058	230	1748	380	2438	530
391	85	1081	235	1771	385	2461	535
414	90	1104	240	1794	390	2484	540
437	95	1127	245	1817	395	2507	545
460	100	1150	250	1840	400	2530	550
483	105	1173	255	1863	405	5060	1100
506	110	1196	260	1886	410	7590	1650
529	115	1219	265	1909	415	10120	2200
552	120	1242	270	1932	420	12650	2750
575	125	1265	275	1955	425	15180	3300
598	130	1288	280	1978	430	17710	3850
621	135	1311	285	2001	435	20240	4400
644	140	1334	290	2024	440	22770	4950
667	145	1357	295	2047	445	25300	5500
690	150	1380	300	2070	450	27830	6050

Exciſe for Common Brewers in *London*, at 5 *s.* *per* Barrel, the Allowances being 2 Barrels in every 22 of Ale.

	0					¼					½					¾				
B.	*l.*	*s.*	*d.*	*f.*	22	*l.*	*s.*	*d.*	*f.*	22	*l.*	*s.*	*d.*	*f.*	22	*l.*	*s.*	*d.*	*f.*	22
1	0	4	6	2	4	0	5	8	0	16	0	6	9	3	6	0	7	11	1	18
2	0	9	1	0	8	0	10	2	2	20	0	11	4	1	10	0	12	6	0	0
3	0	13	7	2	12	0	14	9	1	2	0	15	10	3	14	0	17	0	2	4
4	0	18	2	0	16	0	19	3	3	6	1	0	5	1	18	1	1	7	0	9
5	1	2	8	2	20	1	3	10	1	10	1	5	0	0	0	1	6	1	2	12
6	1	7	3	1	2	1	8	4	3	14	1	9	6	2	4	1	10	8	0	16
7	1	11	9	3	6	1	12	11	1	18	1	14	1	0	8	1	15	2	2	20
8	1	16	4	1	10	1	17	6	0	0	1	18	7	2	12	1	19	9	1	2
9	2	0	10	3	14	2	2	0	2	4	2	3	2	0	16	2	4	3	3	6
10	2	5	5	1	18	2	6	7	0	8	2	7	8	2	20	2	8	10	1	10
11	2	10	0	0	0	2	11	1	2	12	2	12	3	1	2	2	13	4	3	14
12	2	14	6	2	4	2	15	8	0	16	2	16	9	3	6	2	17	11	1	18
13	2	19	1	0	8	3	0	2	2	20	3	1	4	1	10	3	2	6	0	0
14	3	3	7	2	12	3	4	9	1	2	3	5	10	3	11	3	7	0	2	4
15	3	8	2	0	16	3	9	3	3	6	3	10	5	1	18	3	11	7	0	8
16	3	12	8	2	20	3	13	10	1	10	3	15	0	0	0	3	16	1	2	2
17	3	17	3	1	2	3	18	4	3	14	3	19	6	2	4	4	0	8	0	16
18	4	1	9	3	6	4	2	11	1	18	4	4	1	0	8	4	5	2	2	20
19	4	6	4	1	10	4	7	6	0	0	4	8	7	2	12	4	9	9	1	2
20	4	10	10	3	14	4	12	4	2	4	4	13	2	0	16	4	14	3	3	6
21	4	15	5	1	18	4	16	7	0	8	4	17	8	2	20	4	18	10	1	10

Exciſe for Common Brewers in *London*, at 5 *s. per* Barrel, &c. Ale.

Bar.	*l.*	Bar.	*l.*	Bar.	*l.*	Bar.	*l.*
22	5	682	155	1342	305	2002	455
44	10	704	160	1364	310	2024	460
66	15	726	165	1386	315	2046	465
88	20	748	170	1408	320	2068	470
110	25	770	175	1430	325	2090	475
132	30	792	180	1452	330	2112	480
154	35	814	185	1474	335	2134	485
176	40	836	190	1496	340	2156	490
198	45	858	195	1518	345	2178	495
220	50	880	200	1540	350	2200	500
242	55	902	205	1562	355	2222	505
264	60	924	210	1584	360	2244	510
286	65	946	215	1606	365	2266	515
308	70	968	220	1628	370	2288	520
330	75	990	225	1650	375	2310	525
352	80	1012	230	1672	380	2332	530
374	85	1034	235	1694	385	2354	535
396	90	1056	240	1716	390	2376	540
418	95	1078	245	1738	395	2398	545
440	100	1100	250	1760	400	2420	550
462	105	1122	255	1782	405	4840	1100
484	110	1144	260	1804	410	7260	1650
506	115	1166	265	1826	415	9680	2200
528	120	1188	270	1848	420	12100	2750
550	125	1210	275	1870	425	14520	3300
572	130	1232	280	1892	430	16940	3850
594	135	1254	285	1914	435	19360	4400
616	140	1276	290	1936	440	21780	4950
638	145	1298	295	1958	445	24200	5500
660	150	1320	300	1980	450	26620	6050

Excise for Common Brewers in *London*, at 1 *s.* 4 *d.* *per* Barrel, the Allowances being 3 Barrels in every 23 Small Beer.

	0					$\frac{1}{4}$					$\frac{1}{2}$					$\frac{4}{3}$				
B.	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23	*l.*	*s.*	*d.*	*f.*	23
1	0	1	1	3	15	0	1	5	1	13	0	1	8	3	11	0	2	0	1	9
2	0	2	3	3	7	0	2	7	1	5	0	2	10	3	3	0	3	2	1	1
3	0	3	5	2	22	0	3	9	0	20	0	4	0	2	18	0	4	4	0	16
4	0	4	7	2	14	0	4	11	0	12	0	5	2	2	10	0	5	6	0	8
5	0	5	9	2	6	0	6	1	0	4	0	6	4	2	2	0	6	8	0	0
6	0	6	11	1	21	0	7	2	3	19	0	7	6	1	17	0	7	9	3	15
7	0	8	1	1	13	0	8	4	3	11	0	8	8	1	9	0	8	11	3	7
8	0	9	3	1	5	0	9	6	3	3	0	9	10	1	1	0	10	1	2	22
9	0	10	5	1	20	0	10	8	2	18	0	11	0	0	16	0	11	3	2	14
10	0	11	7	0	12	0	11	10	2	10	0	12	2	0	8	0	12	5	2	6
11	0	12	9	0	4	0	13	0	2	2	0	13	4	0	0	0	13	7	1	21
12	0	13	10	3	19	0	14	2	1	17	0	14	5	3	15	0	14	9	1	13
13	0	15	0	3	11	0	15	4	1	9	0	15	7	3	7	0	15	11	1	5
14	0	16	2	3	3	0	16	6	1	1	0	16	9	2	22	0	17	1	0	20
15	0	17	4	0	16	0	17	8	0	16	0	17	11	2	14	0	18	3	0	12
16	0	18	6	2	10	0	18	10	0	8	0	19	1	2	6	0	19	5	0	4
17	0	19	8	2	2	1	0	0	0	9	1	0	3	1	21	1	0	6	3	19
18	1	0	10	1	17	1	1	1	3	15	1	1	5	1	13	1	1	8	3	11
19	1	2	0	1	9	1	2	3	3	7	1	2	7	1	5	1	2	10	3	3
20	1	3	2	1	1	1	3	5	2	22	1	3	9	0	20	1	4	0	2	18
21	1	4	4	0	6	1	4	7	2	14	1	4	11	0	12	1	5	2	2	10
22	1	5	6	0	8	1	5	9	2	6	1	6	1	0	4	1	6	4	2	2

Excife for Common Brewers in *London*, at 1*s.* 4*d.* *per* Barrel, *&c.* Small Beer.

Bar	*l.*	*s.*	*d.*	Bar.	*l.*	*s.*	*d.*	Bar.	*l.*	*s.*	*d.*	Bar.	*l.*	*s.*	*d.*
23	1	6	8	713	41	6	8	1403	81	6	8	2093	121	6	8
46	2	13	4	736	42	13	4	1426	82	13	4	2116	122	13	4
69	4	0	0	759	44	0	0	1449	84	0	0	2139	124	0	0
92	5	6	8	782	45	6	8	1472	85	6	8	2162	125	6	8
115	6	13	4	805	46	13	4	1495	86	13	4	2185	126	13	4
138	8	0	0	828	48	0	0	1518	88	0	0	2208	128	0	0
161	9	6	8	851	49	6	8	1541	89	6	8	2231	129	6	8
184	10	13	4	874	50	13	4	1564	90	13	4	2254	130	13	4
207	12	0	0	897	52	0	0	1587	92	0	0	2277	132	[illegible]	0
230	13	6	8	920	53	6	8	1610	93	6	8	2300	133	6	8
253	14	13	4	943	54	13	4	1633	94	13	4	2323	134	13	4
276	16	0	0	966	56	0	0	1656	96	0	0	2346	136	0	0
299	17	6	8	989	57	6	8	1679	97	6	8	2369	137	6	8
322	18	13	4	1012	58	13	4	1702	98	13	4	2392	138	13	4
345	20	0	0	1035	60	0	0	1725	100	0	0	2415	140	0	0
368	21	6	8	1058	61	6	8	1748	101	6	8	2438	141	6	8
391	22	13	4	1081	62	13	4	1771	102	13	4	2461	142	13	4
414	24	0	0	1104	64	0	0	1794	104	0	0	2484	144	0	0
437	25	6	8	1127	65	6	8	1817	105	6	8	2507	145	6	8
460	26	13	4	1150	66	13	4	1840	106	13	4	2530	146	13	4
483	28	0	0	1173	68	0	0	1863	108	0	0	5060	293	6	8
506	29	6	8	1196	69	6	8	1886	109	6	8	7590	440	0	0
529	30	13	4	1219	70	13	4	1909	110	13	4	10120	586	13	4
552	32	0	0	1242	72	0	0	1932	112	0	0	12650	733	6	8
575	33	6	8	1265	73	6	8	1955	113	6	8	15180	880	0	0
598	34	13	4	1288	74	13	4	1978	114	13	4	17710	1026	13	4
621	36	0	0	1311	76	0	0	2001	116	0	0	20240	1173	6	8
644	37	6	8	1334	77	6	8	2024	117	6	8	22770	1320	0	0
667	38	13	4	1357	78	13	4	2047	118	13	4	25300	1466	13	4
690	40	0	0	1380	80	0	0	2070	120	0	0	27830	1613	6	8

Exciſe for Victuallers in the Country at 5 *s.* *per* Barrel, Strong Beer, and Ale.

B	*l.*	*s.*	*d.*	Bar.	*l.*	*s.*	*d.*
¼	0	1	3	28	7	0	0
½	0	2	6	29	7	5	0
¾	0	3	9	30	7	10	0
1	0	5	0	31	7	15	0
2	0	10	0	32	8	0	0
3	0	15	0	33	8	5	0
4	1	0	0	34	8	10	0
5	1	5	0	35	8	15	0
6	1	10	0	36	9	0	0
7	1	15	0	37	9	5	0
8	2	0	0	38	9	10	0
9	2	5	0	39	9	15	0
10	2	10	0	40	10	0	0
11	2	15	0	50	12	10	0
12	3	0	0	60	15	0	0
13	3	5	0	70	17	10	0
14	3	10	0	80	20	0	0
15	3	15	0	90	22	10	0
16	4	0	0	100	25	0	0
17	4	5	0	110	27	10	0
18	4	10	0	120	30	0	0
19	4	15	0	130	32	10	0
20	5	0	0	140	35	0	0
21	5	5	0	150	37	10	0
22	5	10	0	160	40	0	0
23	5	15	0	170	42	10	0
24	6	0	0	180	45	0	0
25	6	5	0	190	47	10	0
26	6	10	0	200	50	0	0
27	6	15	0				

Exciſe for Victuallers in the Country, at 1 *s.* 4 *d.* *per* Barrel Small Beer.

B.	*l.*	*s.*	*d.*
¼	0	0	4
½	0	0	8
¾	0	1	0
1	0	1	4
2	0	2	8
3	0	4	0
4	0	5	4
5	0	6	8
6	0	8	0
7	0	9	4
8	0	10	8
9	0	12	0
10	0	13	4
11	0	14	8
12	0	16	0
13	0	17	4
14	0	18	8
15	1	0	0
16	1	1	4
17	1	2	8
18	1	4	0
19	1	5	4
20	1	6	8
21	1	8	0
22	1	9	4
23	1	10	8
24	1	12	0
25	1	13	4
26	1	14	8
27	1	16	0
Bar.	***l.***	***s.***	***d.***
28	1	17	4
29	1	18	8
30	2	0	0
31	2	1	4
32	2	2	8
33	2	4	0
34	2	5	4
35	2	6	8
36	2	8	0
37	2	9	4
38	2	10	8
39	2	12	0
40	2	13	4
50	3	6	8
60	4	0	0
70	4	13	4
80	5	6	8
90	6	0	0
100	6	13	4
110	7	6	8
120	8	0	0
130	8	13	4
140	9	6	8
150	10	0	0
160	10	13	4
170	11	6	8
180	12	0	0
190	12	13	4
200	13	6	8

The foregoing Tables (though very ſhort) will ſhew the exact Duty for any Number of Barrels; and their Uſe will be rendered very plain and eaſy, by the Directions and Example following.

1. If the Number of Barrels of ſtrong or ſmall Beer given be leſs than 23, ſeek it in the firſt Column of the Table, and againſt it you have the exact Duty in Pounds, Shillings, Pence, Farthings, and 23d Parts of a Farthing. Thus by the firſt Table, the Duty of 10 Barrels and an half will be found to be 2*l.* 6*s.* 9*d.* 2*f.* and 2 Parts of a Farthing; for againſt 10 in the Column of Barrels, under ½ you have this Sum.

2. For any Number above 23, ſeek it in ſome of the Columns of Barrels in the Table, and if you cannot find the ſame Number, take the next leſs and ſet down the Sum of Money againſt it; then ſubtract the Number found from the Number given, the Remainder (which will ever be leſs than 23) being found in the firſt Column of the Table, gives you a Sum of Money, which added to the former, is the exact Duty required.

EXAMPLE.

EXAMPLE.

What Sum of Money will the Duty of 1306½ *Barrels of ſtrong Beer amount to?*

See the Work by the firſt Table.

			l.	*s.*	*d.*	*f.*	
Number given ———	1306½						
Next leſs *per* Table	1288	=	287	0	0	0	0
Remainder ———	18½	=	4	2	5	1	9
The exact Duty is ———			291 :	2 :	5 :	1 :	9

That is 291 *l.* 2 *s.* 5 *d.* 1 *f.* $\frac{9}{23}$ Parts of a Farthing.

And ſo for any other Number of Barrels by this or any of the other Tables.

SECT. XI.

Short RULES FOR Gauging of Malt.

THE Duty upon Malt is charged upon the Buſhel, and ſo proportionably for a greater or leſſer Quantity. The Buſhel here intended is the *Wincheſter*-Buſhel, which (as ſettled by the firſt Act for laying a Duty upon Malt) contains 2150.42, ſolid Inches, but in Practice 2150, will be ſufficiently near the Truth.

The Area of any plain Figure may be found in Inches, by the Rules laid down in Sect. III. Now if the Area in Inches be divided by 2150, the Quotient ſhews the Buſhels and Parts which ſuch Figure or Veſſel contains at one Inch deep.

Example. There is a Ciſtern or Uting Fat, whoſe Baſe is a Rectangular Parallogram, the Length is 72 Inches, and the Breadth 48 Inches, what is the Area or Content at one Inch deep? *Anſwer*, 1.607, that is one

Buſhel, and .607 thouſand Parts of a Buſhel. For 72 multiplied by 48, is 3456; and this divided by 2150, gives 1.607. But the Area of any ſuch Figure may be more readily found by the Lines A and B upon the Rule. Thus,

As 2150 upon A, is to one of the Sides upon B;
So is the other Side upon A, to the Area upon B;

So in the Example above:

Set 2150 upon A, to 72 upon B;
Then againſt 48 upon A, is 1.607 upon B;
Or Set 2150 upon A, to 48 upon B;
Then againſt 72 upon A, is 1.607 upon B.

The Area thus found, being multiplied by the Mean Depth in Inches, gives the Content of ſuch Ciſtern at that Depth; and ſo for any Quantity of Malt upon the Floor; but Care muſt be taken in finding the mean Depth of the Quantity of Malt, either in the Ciſtern or upon the Floor, by reaſon of the Unevenеſs of the Surface of the Malt, and of the Bottom of the Ciſtern or Floor upon which it is laid; in order to which, take the Depth in 6, 8, or 10 Places; and add all theſe Depths together, and divide the Sum by the Number of Places in which the Depths were taken, the Quotient will be the Mean Depth required.

Example. Suppoſe a Quantity of Malt upon a Floor, of which you take the Depth in Eight ſeveral Places, and find,

The

The	1	Depth to be	4.7	Inches.
	2		5.4	
	3		5.6	
	4		4.9	
	5		4.4	
	6		4.8	
	7		5.3	
	8		5.7	

The Sum of theſe is 40.8, which divided by 8, (the Number of Places) gives 5.1 for the Mean Depth.

Barley is ſometimes ſteeped in Round Veſſels, and the Area of a Circle in Buſhels is thus found: Take the Diameter in Inches, and let the Square thereof be multiplied by .0003643, or divided by 2737.47, the Product or Quotient will be the Area ſought.

Example. Suppoſe the Diameter be 60 Inches, this Square is 3600, and this multiplied by .0003653, or divided by 2737.47, gives 1.315, the Area required.

But for any Round Veſſel, the Rule gives the Area in Buſhels by Inſpection, by help of a certain Number called the *Gauge-Point*, which in this Caſe is 52.32, (being the Diameter of a Circle, whoſe Area is 2150, the Number of ſolid Inches contain'd in the Buſhel.) Thus in the Example above, the Diameter was 60 Inches; therefore ſet the *Gage-Point*, (*viz.* 52.32) upon D, to 1 upon C, then againſt 60 upon D, is 1.315 the Area upon C. And the Rule being thus ſet, againſt any Diameter upon D, you have the Area upon C.

The Diameter and Depth of any Cylindrical Veſſel being given in Inches, the Content in Buſhels, is readily found at once. Thus: Set the *Gauge-point* upon D, to the Depth upon C; then againſt the Diameter upon D, is the Content upon C.

In the laſt Example the Diameter was 60, ſuppoſe the Depth 40 Inches, the Content will be 52.6 Buſhels, and ſo for any other.

Having above declared what the Content of the *Wincheſter*-Buſhel is, I ſhall here, for the Satisfaction of the inquiſitive Reader, give an Account how the Content of the ſaid Buſhel came to be aſcertained.

By the Statute of the 11th *Hen.* VII. *Cap.* 4. It is enacted, that Standard-Weights and Meaſures be made, and ſent to the ſeveral Cities, Burroughs, and Market-Towns therein mentioned, which was accordingly done. But the Statute of the 12th *Hen.* VII. *Cap.* 5. recites, that upon more diligent Examination, the ſaid Standard-Weights and Meaſures were found to be defective. And enacts, That the Meaſure of a Buſhel ſhall contain 8 Gallons of Wheat, and that every Gallon contain 8 Pounds *Troy* of Wheat, and that every Pound contain 12 Ounces *Troy* Weight, and every Ounce contain 20 *Sterlings* [now 20 Pennyweights] and every *Sterling* [or Pennyweight] be of the Weight of 32 Corns of Wheat, that grew in the Middle of the Ear of Wheat. And that a Standard of a Buſhel and a Gallon, after this Aſſize be made, and kept in the King's Treaſury for ever. And new Buſhels and Gallons were ordered to be made, and ſent to the ſeveral Cities, Burroughs, &c. and the old ones brought and broken.

Accordingly there remains now in the Cuſtody of the Chamberlains of the Exchequer a Standard Braſs Buſhel, and a Standard Gallon agreeing thereto. Upon the Buſhel there is this Inſcription,

Henricus Septimus Dei Gratia Rex Angliæ & Franciæ.

In *February* 1696, when a Bill was depending in Parliament, for laying a Duty upon Malt, I did, with the Aſſiſtance of Mr. *George Tollet*, Mr. *Phil. Shales*, and Mr. *Tho. Jett*, and in the Preſence of ſeveral Members of the Houſe of Commons, make an Experiment in order to find the true Content of the ſaid Standard-Buſhel in the Exchequer: And the ſaid Braſs Buſhel being exactly filled with common Spring-Water, and the ſaid Water being meaſured out again by a Regular Parallelepipedon (whoſe Baſe was four Inches ſquare, and the Depth 14 Inches) was found to contain 2145.6 ſolid Inches. The ſaid Water was alſo weighed by the Standard Weights in the Exchequer, and by a Beam which would turn with ſix Grains, with thirty Pounds in each Scale, and found to be 1131 Ounces and 14 Pennyweights *Troy*.

Now to ſettle the Dimenſions for a Buſhel to anſwer the ſaid Standard, theſe were agreed upon as moſt convenient, *viz.* 18.5 Inches Diameter, and 8 Inches deep; for a Cylindrical Veſſel of theſe Dimenſions will contain 2150.42 ſolid Inches, which exceeding the Content of the Standard Buſhel but 4.82 Inches, and there being no other convenient Dimenſions, without counting for the hundredth Part of an Inch, that would come ſo near as theſe: It was enacted, in the Act for laying a Duty upon Malt, *That every Round Buſhel, with a plain and even Bottom, being eighteen Inches and a half Diameter thorughout, and eight Inches deep, ſhall be eſteemed a legal* Winchester-*Buſhel, according to the Standard in his Majeſty's Exchequer.*

When this Experiment was made at the Exchequer, we alſo compared the Standard *Troy* Weights with the Standard *Averdupoiſe* Weights, and found that 15 Pounds of *Averdupoiſe* were equal to 18 Pounds 2 Ounces, and 15 Pennyweight *Troy*; ſo that 140 Ounces *Averdupoiſe* are equal to 173.75 Ounces *Troy*, which in the leaſt Terms in whole Numbers are,

Averd.	*Troy.*
192	235 Ounces.
144	175 Pounds.

The Buſhel, as now ſettled, contains 2150.42 ſolid Inches, as was ſhewn above, and will contain of common Spring-Water 1134,344 Ounces *Troy*. For,

As 2145.6, the Inches in the Braſs Standard, is to
1131.7, the *Troy* Ounces of Water it contained;
So is 2150.42, the Content of a Buſhel, as now ſettled,
to 1134.344.

Hence the following Table ſhews the Content in ſolid Inches, of a Buſhel, half Buſhel, Peck, and Gallon, and likewiſe the Weight of Water that each will contain.

Inches

	Inches.	*Troy.*			*Averdupoiſe.*		
		lb.	oz.	p.w.	lb.	oz	dr.
Buſhel.	2150.42	94.	6.	6.88	77.	12.	8.60
Half Buſh.	1075.21	47.	3.	3.44	32.	14.	4.30
Peck.	537.60	23.	7.	11 72	19.	7.	2 15
Gallon.	268.80	11.	9.	15.86	9.	11.	9 05

By this Table it is eaſy to make any of the Meaſures therein mentioned, or to try ſuch as are already made, by the help of a good Pair of Scales, and true Weight. But where theſe are not to be had, the following Table ſhews the Diameters and Depths that will anſwer to the Buſhel, half Buſhel, Peck, and Gallon, provided they be equally wide from the Top to the Bottom.

Example. To find whether a Buſhel be equal to the Standard or not, ſuppoſe the Diameter be 19 Inches,

Bushel.		Half Bush.		Peck.		Gallon.	
Diam.	Depth.	Diam.	Depth.	Diam.	Depth	Diam.	Depth
17.5	8.94	13.7	7.30	10.7	5.98	8.3	4 97
17 6	8.84	13.8	7.20	10.8	5.87	8.4	4.85
17 7	8.74	13.9	7.09	10.9	5.76	8.5	4.74
17.8	8.64	14.0	6.98	11.0	5 66	8.6	4.63
17.9	8.55	14.1	6 89	11.1	5.56	8.7	4 52
17.0	8 45	14.2	6.79	11.2	5.46	8.8	4.42
18 1	8.36	14.3	6 69	11.3	5.36	8.9	4.32
18.2	8.27	14 4	6.60	11 4	5 27	8.0	4.23
18.3	8.18	14.5	6.51	11.5	5 18	9.1	4.13
18.4	8,09	14.6	6 42	11 6	5.09	9.2	4 04
18.5	8.00	14.7	6.34	11 7	5.00	9 3	3.96
18.6	7.91	14.8	6.25	11.8	4.92	9.4	3.87
18.7	7 83	14.9	6 17	11.9	4 83	9.5	3.79
18.8	7.75	15.0	6 08	12 0	4.75	9.6	3 71
18.9	7 67	15 1	6.00	12.1	4.68	9.7	3.64
19 0	7.58	15 2	5.93	12 2	4 60	9.8	3 56
19.1	7 50	15.3	5.85	12.3	4.52	9.9	3.49
19.2	7.42	15.4	5.77	12 4	4.45	10.0	3.42
19.3	7.35	15.5	5.70	12.5	4.38	10.1	3.36
19 4	7 27	15.6	5.63	12.6	4.31	102	3.29
19 5	7.20	15 7	5.56	12.7	4.24	10.3	3 23

and one tenth Part of an Inch; find this in the Firſt Column; and againſt it you have 7.50; that is, ſeven Inches and a half. Now if the Buſhel be exactly ſo deep, it is a legal Buſhel; if the Depth be more it may be cut to the Depth. If the Buſhel be not exactly round, and the Difference of the Diameters be not above an Inch, add the longeſt and ſhorteſt Diameters together, and take half the Sum for the Mean Diameter, and proceed as above.

Suppoſe the Diameter of a Buſhel be 19.5 Inches, and the Depth but 7 Inches; you will find by the Table, the Depth ſhould be 7.2 Inches; from whence you may conclude this Buſhel is leſs than the Standard. Now, to find how much it wants, ſay,

As 7.2 Inches the Depth in the Table, is to 64, the Pints in a Buſhel,

So is 7 Inches, the Depth of the Buſhel given, to 62.22.

That is 62 Pints, and almoſt a quarter of a Pint; which ſubtracted from 64, the Remainder is 1.77; that is one Pint and three quarters, and ſomewhat better; and ſo much this Buſhel is too little.

Or thus; ſubtract 7, the Depth of the Buſhel given, from 7.2, the Depth found in the Table, the Difference is .2: Then, as 7.2 : 64 : : .2 : 1.777, as above.

The Uſe of the Tables for half Buſhels, Pecks, and Gallons, are the ſame, with reſpect to thoſe Meaſures; and therefore Examples of theſe are needleſs.

The Solidity in Inches of the legal *Wincheſter*-Buſhel, whoſe Dimenſions are as in Page 209, is thus found.

As 1 : 3.14159 : : 18.5 : 58.119415

Half of Circumference is =	29.0597
Half Diameter of the Buſhel	9.25
Product = Area ———	268.8023
Depth of the Buſhel———	8
Content of the Buſhel in Inches	2150.4184

We have ſhewn, how to find the Area of Unity, which is .785398; by which divide Unity, and the Quotient is 1.27324; and if the Content of any Square, as Buſhel, Gallon, *&c.* be multiplied by that, it will give the Diviſors for a Circle in the ſame Meaſure. As for Inſtance, if .2150 the Diviſor for a ſquare Buſhel, be multiplied by 1.27326, the Product is 2737.47 the Diviſor for a Circular Buſhel, and the Square Root of that is 52.32, the Gauge-Point on the Sliding-Rule as marked with M R, ſignifying Malt Round; and the Square Root of 2150 is 46.37, the Gauge-Point for the Square Buſhel; and on the Line D on the Sliding-Rule is marked with M S, ſignifying Malt Square, or the Gage-Point for the ſquare Buſhel of Malt. The Uſe of the Gauge-Point is this, *viz.* in Gauging a ſquare Ciſtern, Couch, or Floor, if they be not truly a Geometrical Square, then find a Mean between the Length and Breadth (as has been ſhewn;) then for the Content in Malt-Buſhels, ſay, *As* the Gage-point on the Line D, 46,37 M S, *Is* to the Depth in Inches, on C,

So is the Geometrical Mean between the Length and Breadth on D,

To the Content in Buſhels, on the Line C.

Example. Admit a Malt-Ciſtern 85 Inches long and 54.4 broad, Depth 36; what is the Content in Malt-Buſhels?

Firſt, the Geometrical Mean between 85 and 54.4 is 68.

Then, by the Rule, $\frac{126}{69}$

	D.		C.		D.		C.
	M S		Depth.		Mean.		Content.
As	46.37	:	36	::	68	:	77.4

The like may be done for a Couch or Floor-Gage.

A Ciſtern, Couch, or Floor-Gage may be more ſpeedily and expeditiouſly caſt up by a Line, which is now put on the Sliding-Rule, marked with M D ſignifying Malt Depth. This Line is no other than a Line of Numbers in a Reverſe Order, and its Uſe is ſo exceeding great for caſting up Malt-Gauges, that to omit it, were to be guilty of one of the greateſt Crimes that could be committed in the whole Book.

On this Line always find the Depth of the Ciſtern, Couch, or Floor. Then ſay, As the Depth upon the Line M D, is to the Length or Breadth upon B; ſo is the Length or Breadth upon A, To the Content in Malt-Buſhels upon B. *Example.* Admit a Floor whoſe Length is 160, Breadth 132, and Depth 8.1 Inches; what is the Content in Malt-Buſhels?

Operation by this Sliding-Rule.

Set the Depth 8.1 on M D, to the Length 160 on the Line A; then againſt the Breadth 132 on the Line A, you 79.5 upon the Line B, the Content in Malt-Buſhels.

Thus,

	M D		B		A		B
As	8.1	:	160	: :	132	:	79.5
	Depth.		*Length.*		*Breadth.*		*Content.*

Or thus,

	M D		B		A		B
As	8.1	:	132	: :	160	:	79.5
	Depth.		*Breadth.*		*Length.*		*Content.*

More Examples in a thing ſo plain were needleſs.

The Duty upon Malt is 6 *d.* *per* Buſhel, but when it is in the Ciſtern, or has been out of it leſs than 30 Hours, the Allowance made to the Maltſter at that time is 4 Buſhels out of 20. According to which Proportion the following Table is calculated.

A TABLE *for the* Duty *of* Malt *when it is in the* Ciſtern, *or has been out of it leſs than* 30 *Hours.*

Buſh	*s.*	*d.*	*qr.*	Buſh.	*l.*	*s.*	*d.*	*qr.*
1	0	4	3,2	34	0	13	7	0,8
2	0	9	2,4	35	0	14	0	0,0
3	1	2	1,6	36	0	14	4	3,2
4	1	7	0,8	37	0	14	9	2,4
5	2	0	0,0	38	0	15	2	1,6
6	2	4	3,2	39	0	15	7	0,8
7	2	9	2,4	40	0	16	0	0,0
8	3	2	1,6	41	0	16	4	3,2
9	3	7	0,8	42	0	16	9	2,4
10	4	0	0,0	43	0	17	2	1,6
11	4	4	3,2	44	0	17	7	0,8
12	4	9	2,4	45	0	18	0	0,0
13	5	2	1,6	46	0	18	4	3,2
14	5	7	0,8	47	0	18	9	2,4
15	6	0	0,0	48	0	19	2	1,6
16	6	4	3,2	49	0	19	7	0,8
17	6	9	2,4	50	1	0	0	0,0
18	7	2	1,6	51	1	0	4	3,2
19	7	7	0,8	52	1	0	9	2,4
20	8	0	0,0	53	1	1	2	1,6
21	8	4	3,2	54	1	1	7	0,8
22	8	9	2,4	55	1	2	0	0,0
23	9	2	1,6	56	1	2	4	3,2
24	9	7	0,8	57	1	2	9	2,4
25	10	0	0,0	58	1	3	2	1,6
26	10	4	3,2	59	1	3	7	0,8
27	10	9	2,4	60	1	4	0	0,0
28	11	2	1,6	61	1	4	4	3,2
29	11	7	0,8	62	1	4	9	2,4
30	12	0	0,0	63	1	5	2	1,6
31	12	4	3,2	64	1	5	7	0,8
32	12	9	2,4	65	1	6	0	0,0
33	13	2	1,6	66	1	6	4	3,2

Buſh.	l.	s.	d.	qr.	Buſh.	l.	s.
67	1	6	9	1,4	100	2	0
68	1	7	2	0,6	110	2	4
69	1	7	7	3,8	120	2	8
70	1	8	0	2,0	130	2	12
71	1	8	4	1,2	140	2	16
72	1	8	9	0,4	150	3	0
73	1	9	2	3,6	160	3	4
74	1	9	7	2,8	170	3	8
75	1	10	0	1,0	180	3	12
76	1	10	4	0,2	190	3	16
77	1	10	9	3,4	200	4	0
78	1	11	2	2,6	210	4	4
79	1	11	7	1,8	220	4	8
80	1	12	0	0,0	230	4	12
81	1	12	4	3,2	240	4	16
82	1	12	9	2,4	250	5	0
83	1	13	2	1,6	260	5	4
84	1	13	7	0,8	270	5	8
85	1	14	0	0,0	280	5	12
86	1	14	4	3,2	290	5	16
87	1	14	9	2,4	300	6	0
88	1	15	2	1,6	310	6	4
89	1	15	7	0,8	320	6	8
90	1	16	0	0,0	330	6	12
91	1	16	4	3,2	340	6	16
92	1	16	9	2,4	350	7	0
93	1	17	2	1,6	360	7	4
94	1	17	7	0,8	370	7	8
95	1	18	0	0,0	380	7	12
96	1	18	4	3,2	390	7	16
97	1	18	9	2,4	400	8	0
98	1	19	2	1,6	500	10	0
99	1	19	7	0,8	1000	20	0

When the Malt has been out of the Ciſtern 30 Hours, or more, the Allowance made to the Maltſter at that Time is 10 Buſhels in every twenty, according to which Proportion the following Table is calculated.

A TABLE *for the* Duty *of* Malt *when it has been out of the* Ciſtern 30 *Hours or more.*

B.	s.	d.	B.	s.	d.	B.	l.	s.	d.	Bſh	l.	s.	d.
1	0	3	34	8	6	67	0	16	9	100	1	5	0
2	0	6	35	8	9	68	0	17	0	110	1	7	6
3	0	9	36	9	0	69	0	17	3	120	1	10	0
4	1	0	37	9	3	70	0	17	6	130	1	12	6
5	1	3	38	9	6	71	0	17	9	140	1	15	0
6	1	6	39	9	9	72	0	18	0	150	1	17	6
7	1	9	40	10	0	73	0	18	3	160	2	0	0
8	2	0	41	10	3	74	0	18	6	170	2	2	6
9	2	3	42	10	6	75	0	18	9	180	2	5	0
10	2	6	43	10	9	76	0	19	0	190	2	7	6
11	2	9	44	11	0	77	0	19	3	200	2	10	0
12	3	0	45	11	3	78	0	19	6	210	2	12	6
13	3	3	46	11	6	79	0	19	9	220	2	15	0
14	3	6	47	11	9	80	1	0	0	230	2	17	6
15	3	9	48	12	0	81	1	0	3	240	3	0	0
16	4	0	49	12	3	82	1	0	6	250	3	2	6
17	4	3	50	12	6	83	1	0	9	260	3	5	0
18	4	6	51	12	9	84	1	1	0	240	3	7	6
19	4	9	52	13	0	85	1	1	3	280	3	10	0
20	5	0	53	13	3	86	1	1	6	290	3	12	6
21	5	3	54	13	6	87	1	1	9	300	3	15	0
22	5	6	55	13	9	88	1	2	0	310	3	17	6
23	5	9	56	14	0	89	1	2	3	320	4	0	0
24	6	0	57	14	3	90	1	2	6	330	4	2	6
25	6	3	58	14	6	91	1	2	9	340	4	5	0
26	6	6	59	14	9	92	1	3	0	350	4	7	6
27	6	9	60	15	0	93	1	3	3	360	4	10	0
28	7	0	61	15	3	94	1	3	6	370	4	12	6
29	7	3	62	15	6	95	1	3	9	380	4	15	0
30	7	6	63	15	9	96	1	4	0	390	4	17	6
31	7	9	64	16	0	97	1	4	3	400	5	0	0
32	8	0	65	16	3	98	1	4	6	500	6	5	0
33	8	3	66	16	6	99	1	4	9	1000	12	10	0

The preceding Tables are ſo plain that one Example will ſufficiently explain them.

A Malſter is charged with 1500 Buſhels in the Ciſtern and Couch, and 350 on the Floor, how much does his Duty amount to?

Becauſe 1500 cannot be found in the firſt Table therefore it muſt be taken out at twice. Thus,

The Duty of 1000 Buſhels in the Ciſtern, &c. is	20	0	0
of 500 Buſhels is	10	0	0
The Duty of 350 on the Floor is	4	7	6
Anſwer *L.*	34	7	6

SECT. XII.

PROPORTIONS IN A CIRCLE.

IT has been ſhewn that if the Diameter of a Circle be 1, the Circumference will be 3.141592, and and the Area .785398, the Square-Root of .785398 is .886227, and this is the Side of a Square, equal to the Area of a Circle, whoſe Diameter is 1. And the Square-Root of .5, *viz.* .707107 is the Side of the greateſt Square that can be inſcribed in a Circle, whoſe Diameter is 1: So that if the Diameter of a Circle be 1,

the	Circumference	is	3.141593
	Square equal		.886227
	Squ. inſcribed		.707107
	Area ———		.785398

Note further, that if the Circumference of a Circle be 1,

$$\text{The}\left\{\begin{array}{l}\text{Diameter}\\ \text{Sq. equal}\\ \text{Sq. inſcribed}\\ \text{Area}\text{——}\end{array}\right\}\begin{array}{c}\text{will}\\ \text{be}\end{array}\left\{\begin{array}{r}.318310\\ 282095\\ .225079\\ .079577\end{array}\right\}$$

If the Area of a Circle be 1,

$$\text{The}\left\{\begin{array}{l}\text{Circumference}\\ \text{Diameter}\text{——}\end{array}\right\}\text{is}\left\{\begin{array}{r}1.128379,\\ 3.54491,\end{array}\right\}$$

If the Circumference of a Circle be 4,

$$\text{The}\left\{\begin{array}{l}\text{Diameter}\\ \text{Area —}\end{array}\right\}\text{is } 1.27324$$

If the Diameter of a Circle be 4,

$$\text{The}\left\{\begin{array}{l}\text{Circumference}\\ \text{Area}\text{——}\end{array}\right\}\text{is } 12.56637$$

In all theſe Caſes the Area is Square Inches, Feet, Yards, *&c.* the other Terms being Lines, refer to Lineal Inches, Feet, Yards, *&c.* reſpectively. Theſe Things premiſed, I ſhall here add ſome uſeful Queſtions, which may be reſolved either by the Pen or Inſtrument.

1. *The Diameter of a Circle given, to find the Circumference.*

As 1 is to 3.141593, Or,
As .31831 is to 1;
So is the Diameter to the Circumference.

2. *The Circumference given, to find the Diameter.*

As 1 is to .31831, Or,
As 3.141593 is to 1;
So the Circumference is to the Diameter.

3. *The Diameter given, to find the Side of a Square, equal to the Circle.*

As 1 is to .886227, Or,
As 1.128379 is to 1;
So is the Diameter to the Side of a Square equal.

4. *The Side of a Square given, to find the Diameter of a Circle whoſe Area is equal to the Square.*

As 1 is to 1.128379, Or,
As 886227 is to 1;
So is the Side of the Square to the Diameter ſought.

5. *The Circumference given, to find the Side of a Square equal to the Circle.*

As 1 is to .282095, Or,
As 3.5491 is to 1;
So is the Circumference to the Side of the Square required.

6. *The Side of a Square given, to find the Circumference of a Circle equal to the Square.*

As 1 is to 3.54491, Or,
As .282095, is to 1;
So is the Side of the Square to the Circumference ſought.

7. *The Diameter of a Circle given, to find the Side of the greateſt Square inſcribed.*

As 1 is to .707107, Or,
As 1 41421 is to 1;
So is the Diameter to the Side required.

8. *The Side of a Square given, to find the Diameter of a Circle that ſhall circumſcribe the Square.*

As 1 is to 1.41421, Or,
As .707107 is to 6;
So is the Side of the Square to the Diameter ſought.

9. *The Circumference of a Circle given, to find the Side of the Square inſcribed.*

As 1 is to .225079, Or,
As 4.4429 is to 1;
So is the Circumference to the Side required.

10. *The Side of a Square given, to find the Circumference of a Circle that ſhall circumſcribe the Square.*

As 1 is to 4.4429, Or,
As .225079 is to 1;
So is the Side of the Square to the Circumference ſought.

11. *The Diameter of a Circle given, to find the Area.*

As 1 is to .785398, Or,
As 1.27324 is to 1;
So is the Square of the Diameter to the Area required.

12. *The Area of a Circle given, to find the Diameter.*

As 1 is to 1.27324, Or,
As .785398 is to 1;
So is the Area to the Square of the Diameter, whoſe Square-Root is the Diameter ſought.

13. *The Circumference of a Circle given, to find the Area.*

As 1 is to .079577, Or,
As 12.56636 is to 1.
So is the Square of the Circumference to the Area ſought.

14. *The Area of a Circle given, to find the Circumference*

As 1 is to 12.56637, Or,
As .079577 is to 1;
So is the Area to the Square of the Circumference,
whoſe Square-Root is the Circumference required.

Such of theſe Proportions as relate to finding the Area of a Circle by the Diameter or Circumference, being of moſt general Uſe, and requiring in ſome Caſes, more Exactneſs than can be attained by the Numbers here given. I have here added two Tables for that Purpoſe, which are calculated to fixteen Places, and are here printed exactly true to ten Places in the Fractions, only the laſt Figure is made one more, where the eleventh Figure is above five. See Figure 24. Plate III.

A TABLE ſhewing the Area and Circumference of any Circle in Inches, Feet, Yards, &c. to any Diameter in Inches, Feet, Yards, &c. from 1 to 12, or from 10 to 120.

D.	Area.	Circumference.
1.0	.7853981634	3.1415926536
1.1	.9503317777	3.4557519189
1.2	1.1309733553	3.7699111843
1.3	1.3273228961	4.0840704497
1.4	1.5393804003	4.3982297150
1.5	1.7671458676	4.7123889804
1.6	2.0106192983	5.0265482457
1.7	2.2698006922	5.3407075111
1.8	2.5446900494	5.6548667765
1.9	2.8352873699	5.9690260418
2.0	3.1415926536	6.2831853072
2.1	3.4636059006	6.5973445725
2.2	3.8013271108	6.9115038379
2.3	4.1547562844	7.2256631033
2.4	4.5238934212	7.5398225686
2.5	4.9087385212	7.8539816340
2.6	5.3091615846	8.1681408993
2.7	5.7255526112	8.4823001647
2.8	6.1575216010	8.7964594301
2.9	6.6051985542	9.1106186954
3.0	7.0685834706	9.4247779608

D.	Area..	Circumference.
3.1	7.5476763502	9.7389372261
3.2	8.0424771932	10.0530964915
3.3	8.5529859994	10.3672557568
3.4	9.0792027689	10.6814150222
3.5	9.6211275016	10.9955742876
3.6	10.1787601976	11.3097335529
3.7	10.7521008569	11.6238928183
3.8	11.3411494795	11.9380520836
3.9	11.9459060653	12.2522013490
4.0	12.5663706144	12.5663706144
4.1	13.2025431267	12.8805298797
4.2	13.8544236023	13.1946891451
4.3	14.5220120412	13.5088484104
4.4	15.2053084423	13.8230076758
4.5	15.9043128088	14.1371669412
4.6	16.6190251375	14.4513262065
4.7	17.3494454294	14.7654854719
4.8	18.0955736847	15.0796447372
4.9	18.8574099032	15.3938040026
5.0	19.6349540849	15.7079632679
5.1	20.4282062300	16.0221226333
5.2	21.2371663383	16.3362817987
5.3	22.0618344098	16.6504410640
5.4	22.9022104447	16.9646003294
5.5	23.7582944428	17.2787595947
5.6	24.6300864041	17.5929188601
5.7	25.5175863288	17.9070781255
5.8	26.4207942167	18.2212373908
5.9	27.3397100679	18.5353966562
6.0	28.2743338823	18.8495559215

D.	Area.	Circumference.
6.1	29.2246656600	19.1637151869
6.2	30.1907054010	19.4778744523
6.3	31.1724531052	19.7920337176
6.4	32.1699087728	20.1061929830
6.5	33.1830724035	20.4203522483
6.6	34.2119439976	20.7345115137
6.7	35.2565235549	21.0486707791
6.8	36.3168110755	21.3628300444
6.9	37.3928065594	21.6769893098
7.0	38.4845100065	21.9911485751
7.1	39.5919214169	22.3053078405
7.2	40.7150407905	22.6194671058
7.3	41.8538681274	22.9336263712
7.4	43.0084034276	23.2477856366
7.5	44.1786466911	23.5619449019
7.6	45.3645979178	23.8761041673
7.7	46.5662571078	24.1902634326
7.8	47.7836242611	24.5044226980
7.9	49.0166993776	24.8185819734
8.0	50.2654824574	25.1327412287
8.1	51.5299735005	25.4469004941
8.2	52.8101725068	25.7610597594
8.3	54.1060794765	26.0752180248
8.4	55.4176944093	26.3893782902
8.5	56.7450173055	26.7035375555
8.6	58.0880481642	27.0176968209
8.7	59.4467869876	27.3318560862
8.8	60.8212337735	27.6460153517
8.9	62.2113885227	27.9601746169
9.0	63.6172512352	28.2743338823

Dai.	Area.	Circumference.
9.1	65.0388219109	28.5884931477
9.2	66.4761005500	28.9026524130
9.3	67.9290871522	29.2168116784
9.4	69.3977817178	29.5309709437
9.5	70.8821842466	29.8451302091
9.6	72.3822947387	30.1592894745
9.7	73.8981131941	30.4734487398
9.8	75.4296396127	30.7876080052
9.9	76.9768739946	31.1017672705
10.0	78.5398153397	31.4159265359
10.1	80.1184666482	31.7300858013
10.2	81.7128249199	32.0442450666
10.3	83.3228911548	32.3584043320
10.4	84.9486653531	32.6725635973
10.5	86.5901475146	32.9867228627
10.6	88.2473375393	33.3008821281
10.7	89.9202357274	33.6150413934
10.8	91.6088417787	33.9292006588
10.9	93.3131557933	34.2433599241
11.0	95.0331777711	34.5575191895
11.1	96.7689077122	34.8716784548
11.2	98.5203456165	35.1858377202
11.3	100.2874914842	35.4999969856
11.4	102.0703453151	35.8141562509
11.5	103.8689071093	36.1283155163
11.6	105.6831768658	36.4424747816
11.7	107.5131545875	36.7566340470
11.8	109.3588402715	37.0707933124
11.9	111.2202339187	37.3849525777
12.0	113.0973355292	37.6981118431

The Uſe of the foregoing Table is obvious, for,

1. IF the Diameter of a Circle be Inches, Feet, Yards, &c. not exceeding 12, you have it in the firſt Column to every tenth Part of an Inch, Foot, Yard, &c. And againſt any Diameter in the firſt Column you have the Area in the ſecond Column, and the *Circumference* in the Third: And the Area is always ſquare Inches, Feet, Yards, &c. according as the Diameter or Circumference is given in Inches, Feet, Yards, &c.

2. If the Diameter given be intire Inches, Feet, Yards, &c. not exceeding 120, you have it in the firſt Column, by taking away the Point, for by this Means the Tenths are made Units; but in this Caſe you muſt remove the Point two Places towards the right Hand in the Area, and one Place in the Circumference,

Thus if the Diameter

be 11.6 the	Area is	105.6831768668
	Circumference is	36.4424747816

But if the Diameter.

be 116, the	Area is	10568.31768668
	Circumference is	364 424747816

And for the ſame Reaſon if the Diameter

be 1160, the	Area is	1056831.768668
	Circumference is	3644.24747816

3. If any Area in the Table be multiplied by 4, the Product will be the Area of the Circle of twice the Diameter of the Circle given, and therefore the Area of a Circle, whoſe Diameter is 22.6 (though not found in the Table) may be readily obtained; for the half of 22.6 is 11.3, which found in the firſt Column, the Area againſt it is 100.2874914842, this multiplied by 4, is 401.1499659368, the Area of a Circle whoſe Diameter is 22.6.

4. If the Diameter and Altitude of a Cylinder or Cone be given, multiply the Area againſt the Diameter by the Altitude, the Product will be the Content of the Cylinder; or by one third Part of the Altitude, the Product will be the Content of the Cone.

5. If a Cylinder, Sphere, or Cone, have the ſame Diameter and Altitude, their Contents are as 3.2.1. Suppoſe the Diameter and Altitude 12, the Area againſt 12 is 113.0973355292, this multiplied by

$$\left.\begin{array}{r}12\\8\\4\end{array}\right\}\text{gives}\left\{\begin{array}{r}1357.1680263504\\904.7786842336\\452.3893421168\end{array}\right\}\begin{array}{l}\text{Cylinder.}\\\text{Sphere.}\\\text{Cone.}\end{array}$$

So that any Area multiplied by two Thirds of its Diameter, gives the Solidity of a Sphere of that Diameter.

6. If the Diameter of a Sphere be doubled, the Area againſt the Sum is the Superficial Content of the Sphere. As, let the Diameter be 6, the Double is 12, the Area againſt 12, is 113.0973355292, the ſuperficial Content of a Sphere whoſe Diameter is 6.

A TABLE ſhewing the Area and Diameter of any Circle in Inches, Feet, Yards, &c. to any Circumference in Inches, Feet, Yards, &c. from 1 to 12, or from 10 to 120.

C.	Area.	Diameter.
1.0	.0795774715	.3183098862
1.1	.0962887406	.3501408748
1 2	.1145915590	.3819718634
1.3	.1344859269	.4138028520
1.4	.1559718442	.4456338407
1.5	.1790493110	.4774648293
1.6	.2037183272	.5092958179
1 7	.2299788928	.5411268065
1.8	.2578310078	.5729577951
1.9	.2872746723	.6047887837
2.0	3183098862	.6366197724
2.1	.3509366495	.6684507610
2.2	.3851549623	.7002817496
2.3	.4209648245	.7321127382
2.4	.4583662361	7639437268
2.5	.4973591972	.7957747155
2.6	.5379437077	.8276057041
2.7	.5801197676	.8594366927
2.8	.6238873769	.8912676818
2 9	.6692465357	.9230986699
3.0	.7161972439	.9549296586

C.	Area.	Diameter.
3.1	.7647395016	.9867606472
3.2	.8148733086	1.0185916358
3.3	.8665986651	1.0504226244
3.4	.9199155711	1.0822536130
3.5	.9748240264	1.1140846016
3.6	1.0313240312	1.1459155903
3.7	1.0894155855	1.1777465789
3.8	1.1490986891	1.2095775675
3.9	1.2103733422	1.2414085561
4.0	1.2732395447	1.2732395447
4.1	1.3376972967	1.3050705333
4.2	1.4037465981	1.3369015220
4.3	1.4713874489	1.3687325106
4.4	1.5406198491	1.4005634992
4.5	1.6114437988	1.4323944878
4.6	1.6838592979	1.4642254764
4.7	1.7578653464	1.4960564651
4.8	1.8334649444	1.5278874537
4.9	1.9106550618	1.5598184423
5.0	1.9894367886	1.5915494309
5.1	2.0698100349	1.6233801495
5.2	2.1517748306	1.6552114082
5.3	2.2353311757	1.6870423968
5.4	2.3204700703	1.7188733854
5.5	2.4072185143	1.7507043740
5.6	2.4955495077	1.7825353626
5.7	2.5854720505	1.8143663512
5.8	2.6769861428	1.8461973399
5.9	2.7700917845	1.8780283285
6.0	2.8647889757	1.9098523171

C.	Area.		Diameter.
6.1	2.9610777162		1.9416903057
6.2	3.0589580062		1.9735212943
6.3	3.1584298457		2.0053622830
6.4	3.2594932345		2.0371832716
6.5	3.3621481728		2.0690142602
6.6	3.4663946505		2.1008452488
6.7	3.5722325977		2.1326762374
6.8	3.6796622843		2.1645072260
6.9	3.78868[illegible]4203		2.1963382147
7.0	3.8992961058		2.22816920[illegible]3
7.1	4.0115003406		2.2600001919
7.2	4.1252961249		2.2918311805
7.3	4.2406834587		2.3236621691
7.4	4.3576623419		2.3554931578
7.5	4.4762327745		2.3873245464
7.6	4.4963947565		2.4191551350
7.7	4.7181482880		2.4509861236
7.8	4.8414933589		2.4828171122
7.9	4.9664299992		2.5146481009
8.0	5.0929581789		2.5164790895
8.1	5.2210779081		2.5783100781
8.2	5.3507891867		2.6101410667
8.3	5.4820920148		2.6419720553
8.4	5.6149863923		2.6538030439
8.5	5.7424723192		2.7056340326
8.6	5.8855497955		2.7[illegible]74650212
8.7	6.0232188213		2.7692960093
8.8	6.1624793955		2.8011269984
8.9	6.3033315212		2.8329579870
9.0	6.4457751952		2.8647889[illegible]57

C.	Area.		Diameter.
9.1	6.5898104187		2.8966199643
9.2	6.7354371916		2.9284500529
9.3	6.8826555140		2.9602819415
9.4	7.0314653858		2.9921129301
9.5	7.1818668070		3.0239439187
9.6	7.3338597777		3.0557749074
9.7	7.4874442978		3.0876058960
9.8	7.6426203673		3.1194368846
9.9	7.7993879862		3.1512678732
10.0	7.9577471546		3.1830988618
10.1	8.1176978724		3.2149298505
10.2	8.2792401396		3.2467608391
10.3	8.4423739563		3.2785918277
10.4	8.6070993224		3.3104228163
10.5	8.7734162379		3.3422538049
10.6	8.9413247029		3.3740847935
10.7	9.1108247153		3.4059157822
10.8	9.2819162811		3.4377467708
10.9	9.4545993944		3.4695777594
11.0	9.6288740571		3.5014087480
11.1	9.8047402692		3.5332397366
11.2	9.9821980307		3.5650707253
11.3	10.1612473417		3.5969017139
11.4	10.3418882021		3.6287327025
11.5	10.5241206120		3.6605636911
11.6	10.7079445712		3.6923946797
11.7	10.8933600799		3.7242256684
11.8	11.0803671381		3.7560566570
11.9	11.2689657456		3.7878876456
12.0	11.4591559026		3.8197186342

The

The Uſes of the Table are very plain; for,

1. IF the Circumference of a Circle be given in Inches, Feet, Yards, *&c.* not exceeding 12, you have it in the firſt Column to every tenth Part of an Inch, Foot, Yard, *&c.* from 1 to 12, and from 10 to 120, in intire Inches, Feet, Yards, *&c.* and in either Caſe you have the the Area in the ſecond Column, and the Diameter in the third Column.

Thus if the Circumference of a Circle.

$$\text{be } 10.5, \text{ the} \left\{ \begin{array}{ll} \text{Area is} & 8.7734162379 \\ \text{Diameter is} & 3.3422538049 \end{array} \right.$$

But if the Circumference

$$\text{be } 105, \text{ the} \left\{ \begin{array}{ll} \text{Area is} & 877.34162379 \\ \text{Diameter is} & 33.422538049 \end{array} \right.$$

2. If any Area in this Table be multiplied by 4, you have the Area of a Circle to twice the Circumference of the Circle given. Let 8.9413247029 (the Area againſt 10.6 Circumference) be multiplied by 4, the Product will be 35.7652988116 which is the Area of a Circle whoſe Circumference is 21.2.

3. If the Circumference of a Cylinder be 10.6 Feet, and the Altitude 16 Feet, the Area againſt 10.6 multiplied by 16, gives the ſolid Content of the Cylinder in Feet and ſo of any other.

4. If the Circumference of a Globe or Sphere be given, the Area againſt it multiplied by 4, gives the ſuperficial Content of ſuch a Sphere. Thus if the Circumference of a Sphere be 3 Feet, the Area againſt 3 is

.7161972439; this multiplied by 4, gives 2.8647889756 the ſuperficial Content of a Sphere, whoſe Circumference is 3 Feet. Or thus, double the Circumference given, and find the Sum in the firſt Column, the Area againſt it is the ſuperficial Content ſought; thus twice 3 is 6, which found in the firſt Column, the Area againſt it is 2.8647889756, as above. And indeed, any Area in this Table is equal to the ſuperficial Content of a Sphere, whoſe Circumference is half the Circumference againſt ſuch Area reſpectively.

5. If the Circumference of a Sphere be found in the firſt Column, you have the Diameter in the third Column. Now if the Area between them be multiplied by two Thirds of the Diameter. you have the Solidity of ſuch Sphere.

SECT. XIII.

The Uſe of the Inſtrument in Meaſuring of Timber and Stone.

Of ROUND TIMBER.

PROB. I.

The Circumference and Length of any Round Tree, being given (in Foot-meaſure) to find the Content.

RULE.

AS 3.545 upon D, is to the Length upon C; So is the Circumference upon D, to the Content upon C.

Example. Suppoſe the Length of a Tree be 20 Feet, and the Circumference 4 Feet. What is the Content ?

Set 3.545, upon D, to 20 (the Length) upon C, then againſt 4 the Circumference upon D, you have 25.46 upon C, that is 25 Feet and almoſt an half, and ſo much is the Content.

Note, 3.545, is the Circumference of a Circle whoſe Area is 1.

PROB.

PROB. II.

The Circumference of a Round Tree being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Rule given in the laſt Problem will ſerve here, if inſtead of 3.545, you make uſe of 42.54 (which is found by multiplying 3.545 by, 12.)

Example. The Circumference is 36 Inches, and the Length 10 Feet, What is the Content ?

Set 42.54, upon D, to 10 upon C, and againſt 36 upon D, is 7.16 upon C, which is the Content ſought.

PROB. III.

The Circumference of any Round Tree being given, to find how much in Length it will take to make a ſolid Foot.

FIrſt in Foot-Meaſure the Proportion is,

As the Circumference upon D, is to 1 upon C ;
So is 3.545 upon D, to the Length ſought upon C.

Example. If the Circumference of a Tree be 3 Feet, How many Feet in Length will it take to make a ſolid Foot ?

Set 3 upon D, to 1 upon C, and againſt 3.545 upon D, is 1.396 upon C, that is 1 Foot and .396 Parts. Now, ſo many times as this is contained in the Length of the Tree, ſo many Feet does the Tree contain.

Secondly, In Inch-meaſure, if the Circumference be given in Inches, ſay,

As the Circumference upon D, is to 12 upon C;
So is 42.54 upon D, to the Length in Inches (upon C) to make a Foot.

Example. The Circumference of a Tree is 48 Inches; How many Inches in Length will make a Foot?

Set 48 upon D, to 12 upon C, and againſt 42.54 upon D, is 9.42 upon C, that is 9 Inches and .42 Parts, the Length required.

Now ſuppoſing this Tree were 20 Feet long, the Content will be found to be 25.47, that is, 25 Feet, and almoſt an half; for 20 Feet is 240 Inches, and 9.42 Inches are contained 25.47 times in 240.

Note, The common Way uſed for the Meaſuring of Round Timber, is to girt the Tree about in the Middle with a Line, and to take the fourth Part of the Circumference for the Side of a Square equal to the Circle; but this is falſe, though moſt Men uſe it; for according to this Suppoſition, it will take 12 Inches in Length of the Tree above-mentioned to make a Foot, and ſo the whole Tree would contain 20 Feet, whereas the true Content is 25.47 Feet as ſhewn above.

Note, In all that hath been ſaid of Round Timber, we ſuppoſe the Tree to be equally thick from End to End. But ſeeing moſt Trees are leſs towards the Top than at the Root, the Circumference muſt be taken in the Middle, and if the Ends differ much, it will be the more exact, if you divide the Length into two equal Parts, and take the Circumference in the Middle of each, and ſo find the Content as if they were two Trees. Theſe Contents added together give the Content of the whole.

For Inſtance, There is a Tree of 12 Feet long, the Circumference at one End is 6 Feet, and at the other End 3 Feet, the Sum of theſe is 9, the half is 4.5 the Circumference in the Middle, by which (and the Length) the Content will be found (by Prob. I.) to be 19.34 Feet.

But if this Tree be divided into Parts, as each may be 6 Feet long, the Circumference in the Middle of the greater will be 5.25, by which the Content will be found to be 13.16 Feet.

And the Circumference in the Middle of the leſſer Piece will be 3.75, and the Content of this will be 6.72, which added to the Content of the other, *viz.* 13.16, gives 19.88 Feet, the Content of the whole Tree: Whereas the Content found by one Circumference taken in Middle of the whole Tree, is but 19.34.

But in moſt Caſes it will be ſufficient to take the Circumference in the Middle of the Tree, and ſo find the Content, as in Prob. I.

Obſerve alſo, in taper Trees, when it is ſaid ſo much in Length will make a Foot, it muſt be underſtood in that Place only where the Circumference was taken; for it will not take ſo much in Length of the greater End of a Tree to make a Foot, as it will of the leſſer.

Yet, if by the Circumference taken in the middle you find the Length of a Foot, that Length ſo found will ſhew the whole Content of the Tree, as in Prob. III.

PROB. IV.

The Circumference of any Round Tree being given, either in Feet or in Inches, to find how many Solid Feet are contained in one Foot of the Length.

If the Circumference be given in Feet, ſet 3.545 upon D, to 1 upon C; then againſt the Circumference upon D, is the Content upon C.

Example. Let the Circumference of a Round Tree be 4 Feet, How many Solid Feet are contained in one Foot of the Length of this Tree? Anſwer, 1.273:

For, As 3.545 upon D is to 1 upon C;
So is 4 upon D, to 1.273 upon C.

If the Circumference be given in Inches, make uſe of 42.54, inſtead of 3.545, and then the Proportion will hold as above.

Of Square Timber.

PROB. V.

The Length and Side of any Piece of Square Timber being given in Foot-Meaſure, to find the Content.

The Proportion is,

As 1 upon D, is to the Length upon C;
So is the Side upon D, to the Content upon C.

Example. There is a Piece of Timber exactly Square, each Side at either End being 1.5 Foot, and the Length 20 Foot, How many Solid Feet doth this Piece contain?

Set 1 upon D, to 20 upon C, and againſt 1.5 upon D, is 45 upon C, which is the Content ſought.

PROB. VI.

The Side of the Square Solid being given in Inches, and the Length in Feet, to find the Content in Feet.

THE Proportion in the laſt Problem will hold here, if inſtead of 1 you make uſe of 12 for the firſt Term.

Example. Let the Side be 18 Inches, and the Length 20 Feet, what is the Content in Feet?

Anſwer 45: For,

As 12 upon D, is to 20 the Length upon C;
So is 18 the Side upon D, to 45 the Content upon C.

When the Ends are not exactly ſquare (which often happens) take the Breadth and Thickneſs at the End in Feet and Parts; then multiply the Breadth by the Thickneſs, and this Product by the Length, the laſt Product will be the Content.

If the Breadth and Thickneſs be given in Inches, and the Length in Feet, then by the Lines A and B on the Rule,

The Proportion is,

(1). As 1 upon B, to to the Breadth upon A;
So is the Thickneſs upon B, to a fourth Number, *viz.* (the Area of the End in ſquare Inches.)

(2). As 144 upon B, to the fourth Number upon A;
So is the Length upon B, to the Content ſought.

But if any ſuch Solid be greater at one End than at the other, the exact way to find the Content, is to find the Area of each Baſe (in Foot-Meaſure) then find a Geometrical Mean betwixt the ſaid Area's; then if the Sum of theſe three be multiplied by one third Part of the Length, the Product is the Content.

SECT. XIV.

The Uſe of the Rule in Menſuration of Brick-work.

BRICK-work is commonly meaſured by the Rod-Square, which is 16.5 Feet in Length, and as much in Breadth, and by conſequence each ſquare Rod contains 272.25 ſquare Feet; for 16.5 multiplied by 16.5 is 272.25.

Note alſo, all Brick-work muſt be reduced to the Standard-meaſure of one Brick and an half thick.

PROB. I.

The Length and Height of any Brick-Wall being given in Feet, to find the Content in ſquare Rods at any Thickneſs.

The RULE.

MUltiply the Height by the Length, and divide the Product by 272.25, the Quotient will be the true Content, if the Wall be juſt one Brick and an half thick:

If the Thickneſs be more or leſs; Then ſay,

As 3 is to the Number of half Bricks in the Thickneſs of the Wall;
So is the Quotient above found to the true Content ſought.

Or having the Length and Height in Feet, you may find the Content more readily on the Rule by this Proportion.

As 272.25 upon B, is to the Height upon A ;
So is the Length upon B to a fourth Number upon A, which is the true Content, if the Wall be one Brick and an half thick : If the Thickneſs be more or leſs, ſay,

As 3 upon B is to the half Brick thick upon A ;
So is the fourth Number above found upon B, to the Content required.

Example. There is a Wall 16 Feet high, and 27 Feet long : How many ſquare Rod doth this Wall contain at one Brick and an half thick ? Anſwer 5 Rod and 7 tenth Parts : For,

As 272.25 upon B, is to 16 the Height upon A ;
So is 97 the Length upon B, to 5.7 A.

Now if the Wall above-mentioned were two Bricks and an half thick, it would contain 9 Rod and an half : For,

As 3 upon B, is to 5 (the half Bricks in two and a half) upon A ;
So is 5.7 the Content at one Brick and an half thick upon B, to 9.5 the Content ſought upon A.

But if a Wall of 16 Feet high, and 97 Feet long were but 1 Brick thick, the Content would be but 3 Rod and 8 tenth Parts : For,

As 3 upon B, is to 2 (the half Bricks in one) upon A;
So is 5 7 (the Content at one and an half thick) upon B, to 3.8 the Content upon A.

Now, that (in all theſe Caſes) you may find the Content at one Operation: I ſhall here lay down certain fix'd Numbers for any Thickneſs hereafter expreſſed,

viz. at	1	Bricks thick	408.37
	1½		272.25
	2		204.19
	2½		163.35
	3		136.12
	3½		116.68
	4		102.10
	4½		90.75
	5		81.68

By theſe Numbers the Content of any Wall, at any Thickneſs here mentioned may be found either by the Pen or Rule, and the Proportion is,

As the Number proper to the Thickneſs, is to the Height;
So is the Length to the Content.

Example. There is a Wall four Bricks and an half thick, 7 Foot high, and 85 Foot long: How many ſquare Rod doth this Wall contain?

The proper Number for four Bricks and an half thick, is 90.75.

Therefore, Set 90.75 upon A, to 7 the Height upon B; then againſt 85 the Length upon A, is 6 555 upon B; that is, ſix Rod and an half, and ſomething more.

Note, The decimal Parts of a Rod may be reduced into Feet by this Proportion.

As 1.00 upon A, is to 272.25 upon B;
So is any Decimal upon A, to the Number of Feet
upon B.

And thus you may find the Decimal in the laſt Example, *viz.* .555 is equal to 151 Feet.

Alſo	¼ or .25	of a Rod is	68	Feet
	½ or .50		136	
	¾ or .75		204	

PROB. II.

The Height of any Wall being given in Feet, to find how many Feet in Length it will take to make a ſquare Rod at any Thickneſs abovementioned.

RULE.

SET the Height upon A, to the Number proper for the Thickneſs upon B; then againſt 1 upon A, is the Length ſought upon B.

Example. There is a Wall one Brick and an half thick, and 8 Feet high: How many feet in Length will it take to make a ſquare Rod? Anſwer, 34. For,

The proper Number for one and an half thick is 272.25. Therefore,

As 8 (the Height) upon A, is to 272.25 upon B;
So is 1 upon A, to 34 upon B.

SECT. XV.

The Uſe of the Rule in the Menſuration of Superficies, as Board, Paving, Cieling, Wainſcotting, Flooring, Tyling, &c.

NOTE, In all Square or Oblong Superficies the Area or Content may be found by this Proportion:

As 1 upon A, to the Breadth upon B,
So is the Length, upon A, to the Content upon B.

This Proportion will hold in Triangular Superficies alſo, if you take half the Perpendicular for the Breadth, and the Side upon which it falls for the Length: And in all Caſes, if the Sides be given

in	Inches	the Area	Inches,
	Feet	will be	Feet or
	Yards	Square	Yards.

But in many Caſes the Sides are given in one kind of Meaſure, and the Area required in another: I ſhall therefore give ſome Examples of each.

I. *For Board.*

IF the Length and Breadth be both given in Inches, and the Content ſought in Feet. Say,

As 144, is to the Breadth;
So is the Length, to the Content.

Example. A Board is 14 Inches broad, and 156 Inches long: How many ſquare Feet doth it contain? Anſwer 15.16. For,

As 144 upon A, is to 14 the Breadth upon B;
So is 156 the Length upon A, to 15.16 the Content upon B.

If the Breadth be given in Inches, and the Length in Feet, ſay,

As 12, is to the Breadth;
So is the Length, to the Content.

Example. A Board is 14 Inches broad, and 13 Foot long: What is the Content in Feet? Anſwer 15.16. For,

As 12 upon B. is to 14 the Breadth upon A;
So is 13 the Length upon B, to 15.16 the Content upon A.

II. *Of Paving.*

PAvier's Work is meaſured by the Yard Square, which contains 9 ſquare Feet: The moſt natural way is to meaſure the Sides with a Yard divided into 100 equal Parts, and then multiply the Breadth by the Length, the Product will be the Content in ſquare Yards: But if the Sides be given in Feet, Say,

As 9 is to the Breadth;
So is the Length to the Content.

Example. There is a Court paved with Stone, the Breadth is 64 Feet, and the Length 95 Feet: What is the Content in ſquare Yards? Anſwer 67.55. For,

As 9 upon A, is to 64 the Breadth upon B;
So is 95 the Length upon A, to 67.55 the Content upon B.

III. *Of Cieling.*

CIeling and alſo Wainſcotting, are meaſured by the Yard Square, and therefore what is ſaid in the Example above, may be underſtood here.

IV. *Of Flooring and Tyling.*

THeſe are both meaſured by the Square, which contains 100 ſquare Feet, the Side being 10 Feet: Therefore if the Length and Breadth be given in Feet, multiply the one by the other, and cut off two Figures in the Product, the Remainder ſhews the Content in Squares, and the Figures cut off are the Parts of a Square, or odd Feet.

Or by the Rule, ſay,

As 100, is to the Breadth in Feet;
So is the Length in Feet, to the Content in Squares.

Example. There is a Floor 66 Feet broad, and 75 Feet long. How many Squares doth it contain? Anſwer, 49.5. For,

As 100 up A, is to 66 the Breadth upon B:
So is 75 the Length upon A, to 49.5 the Content upon B.

The like for Tyling.

Laſtly. The Breadth of any Superficies being given in Feet or Yards, to find how many in Length (of either) it will take to make a Foot or Yard ſquare.

The Proportion is,

As the Breadth is to 1;
So is 1, to the Length ſought.

Example. A Board is 1.5 Foot broad: How much in Length will it take to make a Foot ſquare? Anſwer, .667. For,

As 1.5, (the Breadth) upon A, is to 1 upon B;
So is 1 upon A, to .667 upon B.

So that if one Side of a Piece of Board be 1.5 Foot, the other Side muſt be .667 of a Foot to make a ſquare Foot; for .667 multiplied by 1.5 is 1.

In Inch-Meaſure the Proportion is,

As the Breadth in Inches, is, to 12;
So is 12
To the Length in Inches to make a Foot.

And, As the Breadth in Inches, is to 36;
So is 36
To the Length in Inches to make a Yard.

Example. A Board is 18 Inches broad: How many Inches in Length will make a Foot?

Set 18 (the Breadth) upon A, to 12, upon B, then againſt 12 upon A, is 8 upon B; and ſo many Inches in Length will make a Foot of this Board; for 18 multiplied by 8, gives 144 the ſquare Inches in a ſquare Foot.

APPENDIX.

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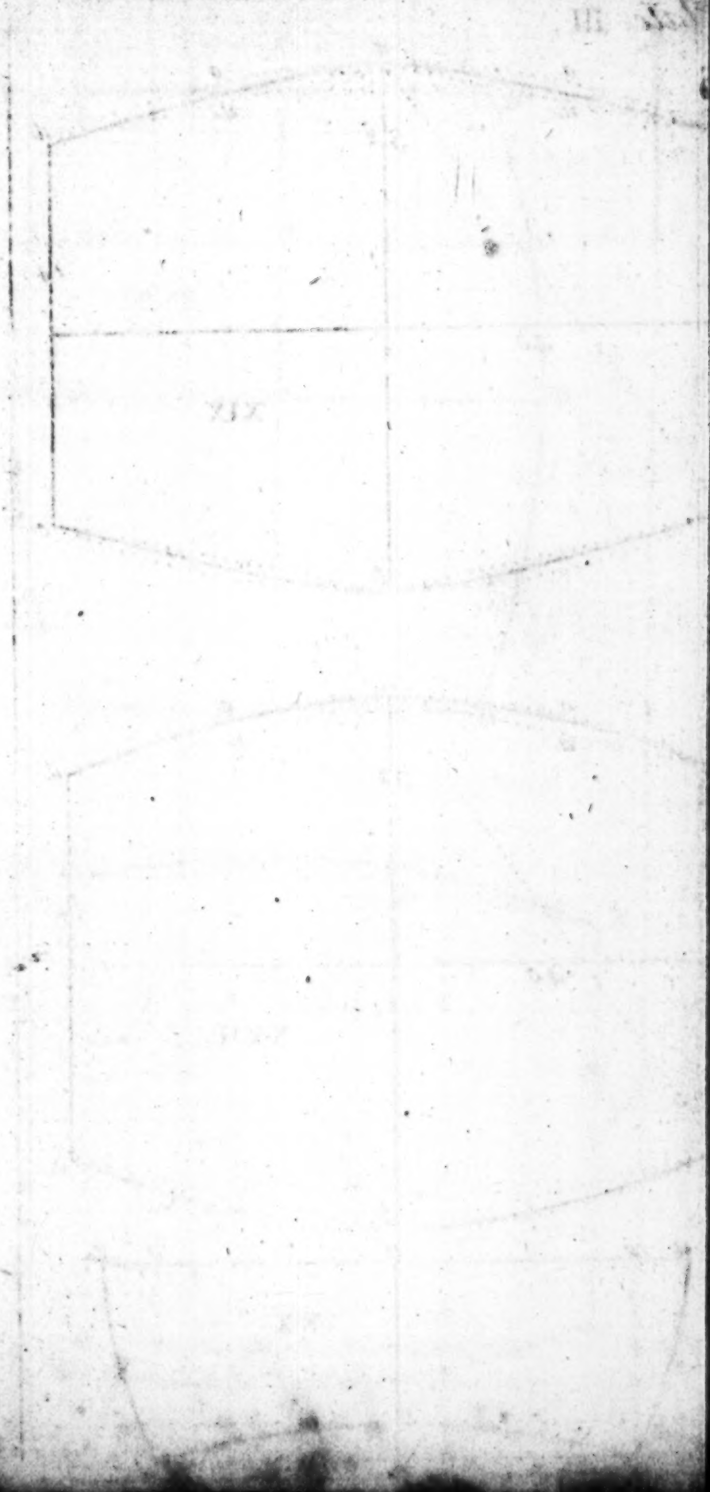
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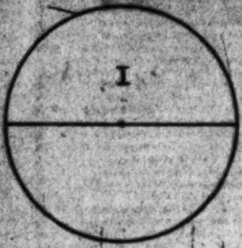

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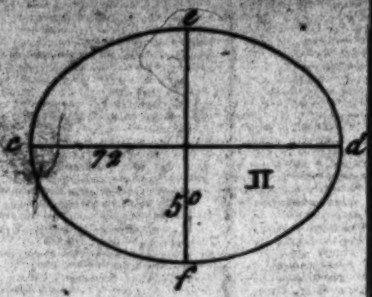

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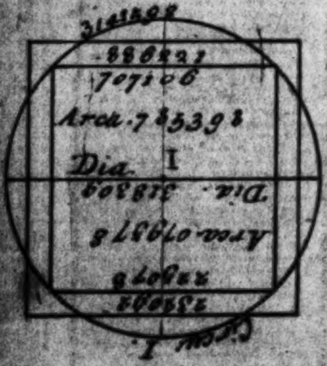

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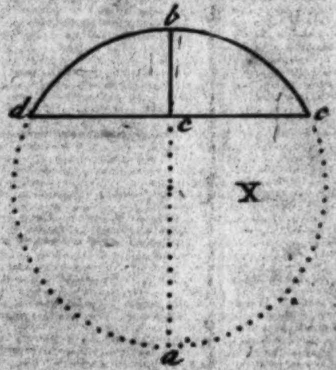

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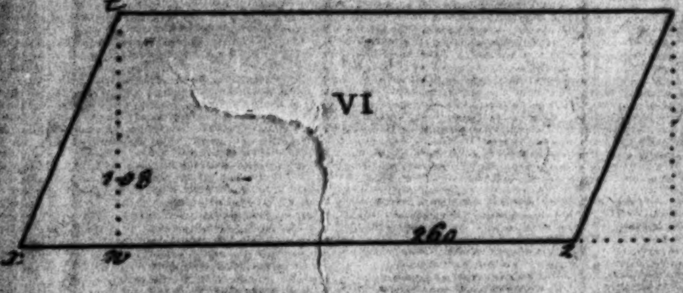

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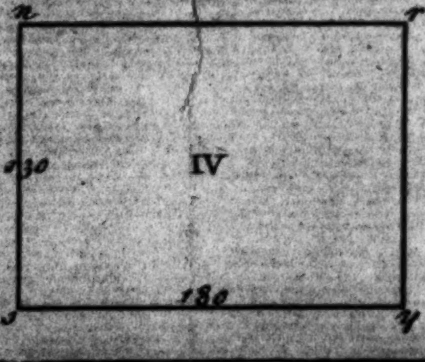

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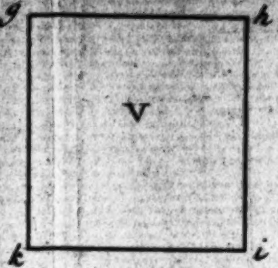

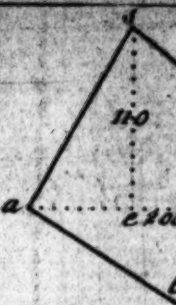

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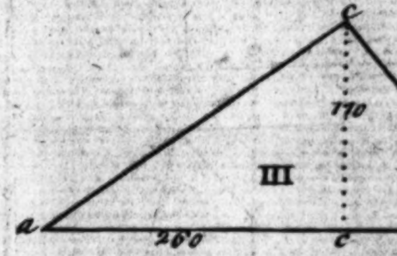

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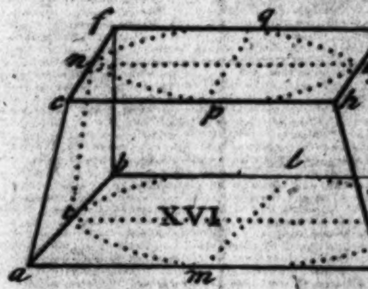

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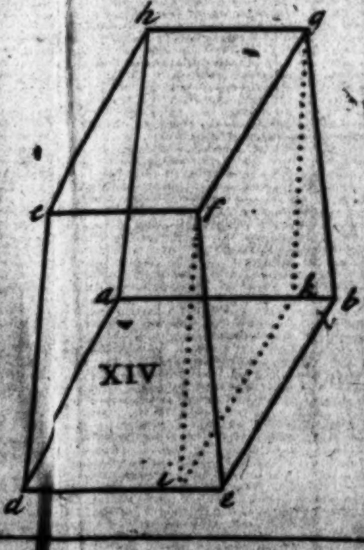

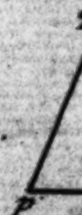

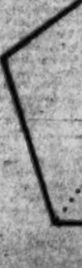

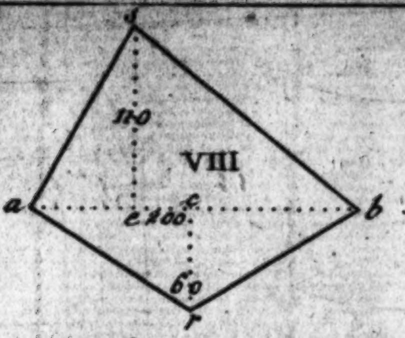
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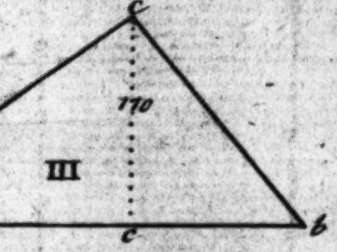
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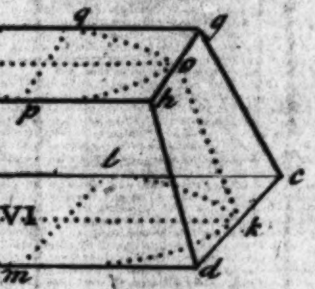
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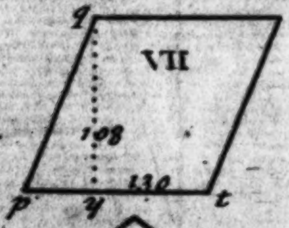
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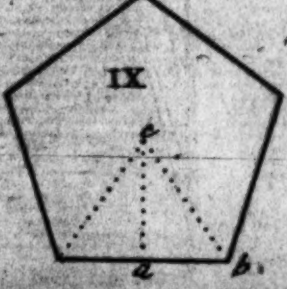
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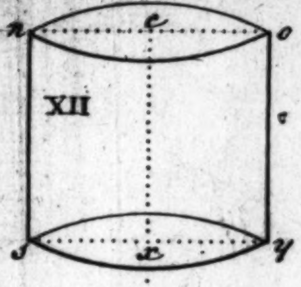
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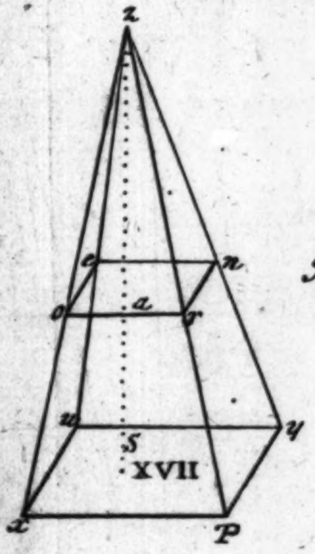
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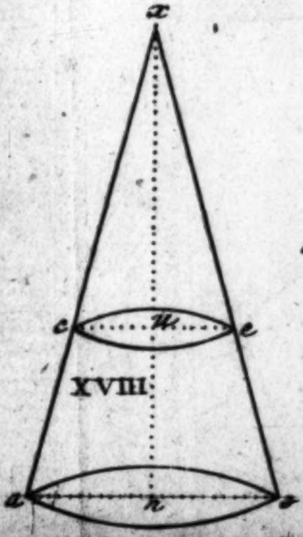
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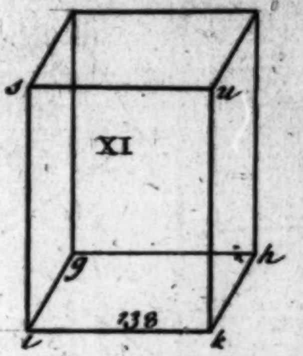
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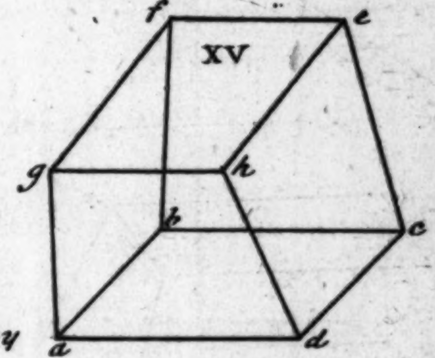
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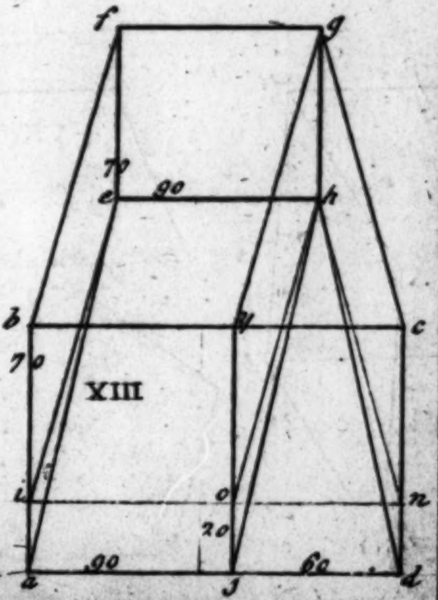
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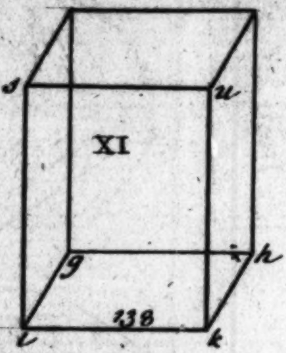

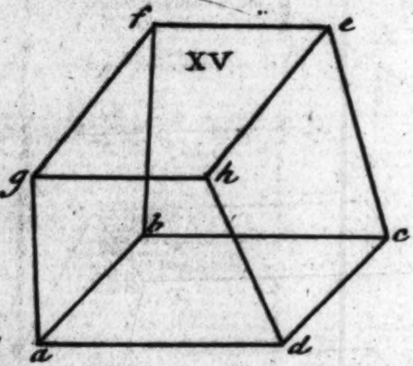

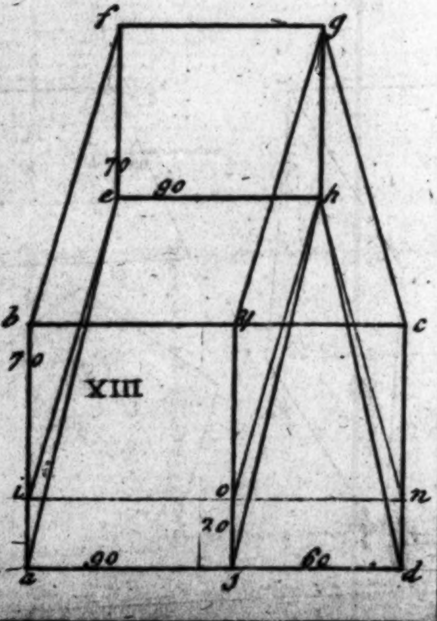

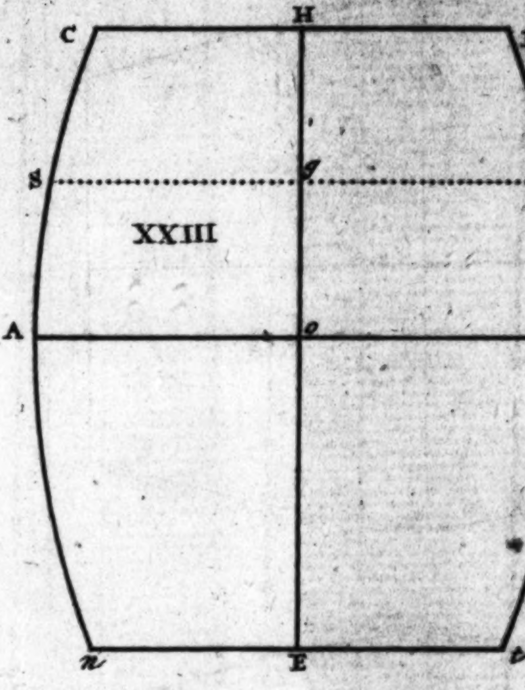

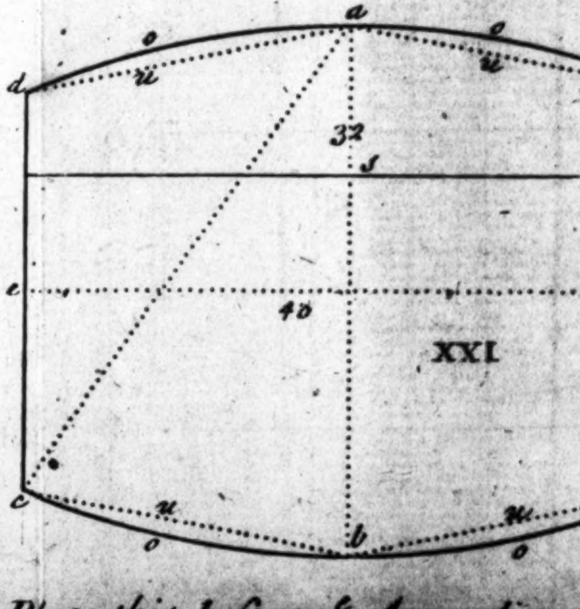

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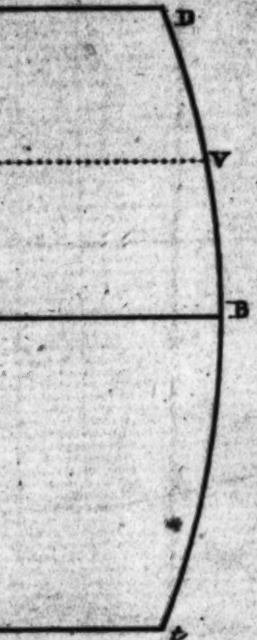
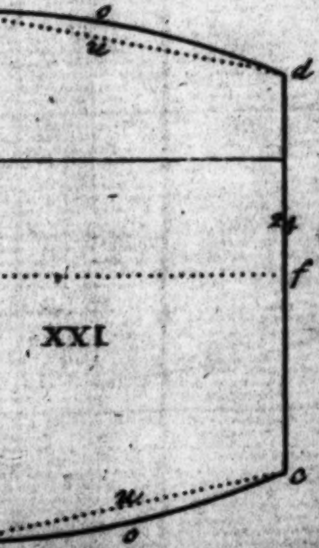
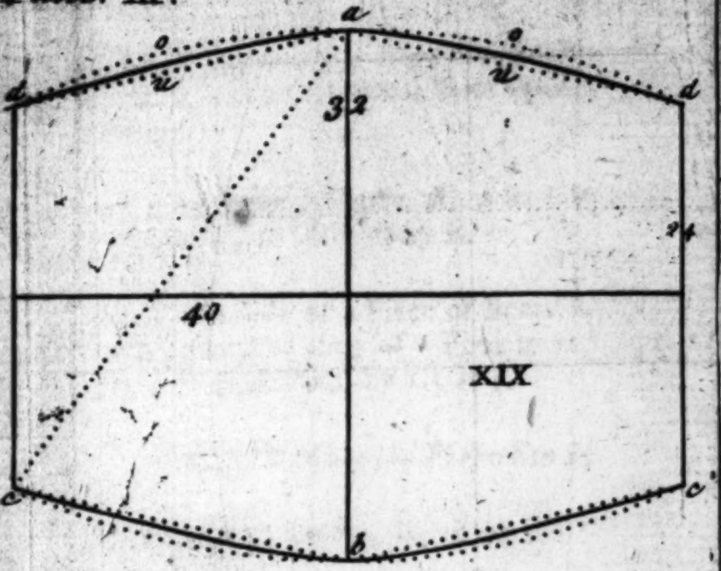
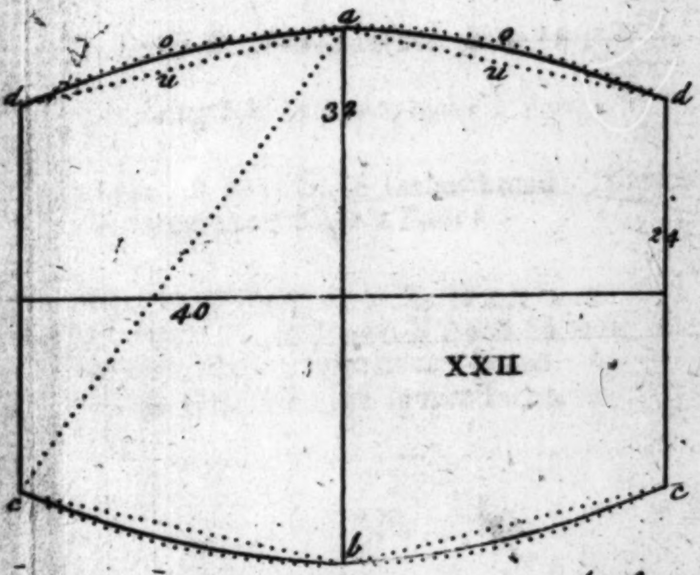


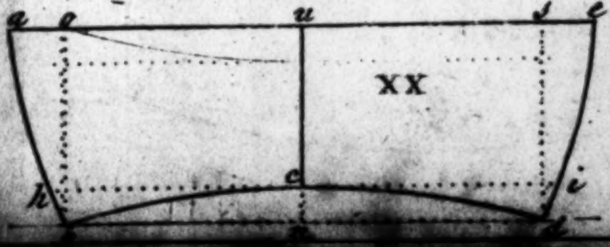


Appendix.

Plate III.







APPENDIX.

Of Conic Sections.

IN the Doctrine of Conics there are theſe two Things (very different) to be ſeparately conſidered. *Firſt*, what Figures or Curve-Lines ariſe from the Section of a Cone by a Plain. And *Secondly*, What is the Nature of ſuch Figures or Curve-Lines, conſidered without reſpect had to ſuch ſuppoſed Original, by cutting a Cone.

See Dr. *Wallis de Con. Sect.* and Treatiſe of *Algebra*, p. 292.

DEFINITION.

1. *A* Cone *is a Solid deſcribed, when one of the containing Sides of a Rectangled Triangle remaining fixed, the Triangle is turned round about till it return to the Place from whence it firſt moved: The Axis of a* Cone *is that fixed Side about which the Triangle is moved: The Baſe of the* Cone *is a Circle, which is deſcribed by the containing Side that is moved about.*

Of the ſeveral Sections of a Cone.

1. IF a Cone be cut by a Plain through its Axis, from the Vertex to the Center of the Baſe, the Section will be a Triangle as S U B. Fig. 1 and 2. Plate IV.

2. If a Cone be cut by a Plain at Right Angles to its Axis, (or if a Right Cone, Parallel to its Baſe) the Section will be a Circle, whoſe Diameter let be S *d* B. Fig. 1.

3. If a Cone be cut by a Plain Parallel to the Side of the Triangle (S V) the crooked Section C *c c* N *c c* C will be a Parabola; in which P N, is the Diameter, or Axis, and *dc* an Ordinate rightly applied. Fig. 1.

4. If a Cone be cut by a Plain that will alſo cut one of the Sides of the Triangle produced in E. (Fig. 1.) the crooked Section O *o o* and A *o o* O is an Hyperbola, in which A H is the intercepted Diameter: A E the Tranſverſe Diameter; and *b o* an Ordinate rightly applied.

5. If a Cone be by a Plain through both the Sides of the Triangle under the Vertix V, the crooked Section, *a o o* A *o o* will be an Ellipſis, in which *a* A is the Diameter, and D *o*, an Ordinate rightly applied. Fig. 2.

Theſe are all the Sections that can be made by cutting a Cone by a Plain. I ſhall treat only of the four laſt, and of theſe but ſo much as I conceive neceſſary to my preſent Deſign, which is only to explain the

Nature and Original of ſome Solids, which are frequently mentioned in Books, and ſometimes occur in the Practice of Gauging.

Of a Parabola.

IN the Parabola (Fig. 1.) Let the Cone be cut by a Plain (S *d* B) Parallel to the Circular Baſe (S P B) it will appear,

1. That S *d* is equal to S P, for N P is parallel to S V, and S *d* B, is parallel to S P B.

2. That S P multiplied by P B is equal to P C ſquared. Therefore S *d* × *d* B = *d c q*.

3. As N P, is to P C ſquared ;
So is N *d*, to *d c* ſquared. Therefore,

4. As N P, is to N *d*.
So is P C ſquared, to *d c* ſquared, which is the general Property of a Parabola.

PROB. I. Fig. 3. Plate IV.

To deſcribe a Parabola in Plano.

DRAW the ſtraight Line O P O, and upon the middle Point P, erect the Perpendicular P A ; compleat the Triangle by drawing the Lines A O, A O ; then through A P (the Axis) and parallel to O P O (the Baſe) draw a competent Number (the more the better) of ſtraight Lines sDs. Now whereſoever D be taken, it will be

$AP : PO :: AD : De$. And
$PO \times De = Doq$.

This gives the Points o, o (as many as you pleaſe on both Sides) through which the Curve Line $O\,o\,o\,A\,o\,o\,O$ being drawn, you have the Parabola required.

Or thus;

Having drawn the Parallellogram ONVO, (Fig. 3.) divide AN into any Number of equal Parts, and ſuppoſe AP divided into as many equal Parts as the Square of the Number of Parts in AN, as here AN is divided into 4 equal Parts, and AP is ſuppoſed to be divided into 16 equal Parts. Let Lines be drawn from N n, n, n, parallel to AP. Now, if from a Scale of equal Parts, of which 16 are equal to AP, you make the firſt $nc = 1$, the ſecond $nc = 4$, the third $nc = 9$, and the fourth $NO = 16$, the Points c, c, c, O, are in the Parabola, and by a competent Number of Points thus found, the *Parabola* may be drawn.

Note, 1. The Parabola $OcAO$, is two thirds of the circumſcribing Parallelogram ONVO, and the Triangle OAO, is three Fourths of the Parabola; therefore if the Parallelogram be 3, the Parabola is 2, and the Triangle 1.

2. If the Parallelogram PANO, being turned about the Line AP, it will generate the Cylinder

ONVO, and by the ſame Converſion there will be generated by the Curve-Line O *c c c*, the Parabolic Conoid, O *c* A *o o*, and by the ſtraight Line O A, the Cone O A O, and theſe three Solids are as 3, 1 and a half, and one; for the Parabolic Conoid, is half the circumſcribing Cylinder, and the Cone one third Part thereof. Now, all theſe Properties being common to every Parabola (as is demonſtrated by Dr. *Wallis* in his *Mechanica*, Part II. Chap. V.) the Content of any Parabolic Conoid is eaſily found, as ſhall be ſhewn in the next Problem.

PROB. II.

There is a Parabolic Conoid (Fig. 3.) O A O, *the Diameter of the Baſe is* 32 *Inches, and* A P, *the Altitude* 45.7143 *Inches: What is the Content in Ale-Gallons?*

Multiply the Area of the Baſe by half the Altitude, the Product gives the Content ſought.

Example. The Diameter is 32. The Area is 2.8519, this multiplied by half the Altitude (*viz.*) 22.1571 gives 65.18 the Content ſought.

Or, Multiply the Square of the Diameter by half the Altitude, and that Product by .0027851, the laſt Product will be the Content in Gallons.

PROB. III.

There is a Fruſtum of a Parabolic Conoid, (Fig. 3.) *whoſe Baſes are Parallel;* O P O, *the Diameter of the greater Baſe is* 32 *Inches:* *c* D *o*, *the Diameter of the leſſer Baſe* 24 *Inches, and* P D *the Altitude* 20 *Inches: What is the Content in Ale-Gallons?*

FROM the Square of P O (the Semidiameter of the greater Baſe) ſubtract the Square D O (the Semidiameter of the leſſer Baſe) then ſay,

As that Difference,
Is to P D the Altitude of the Fruſtum;
So is the Square of P O,
To P A the Axis of the whole Conoid.

In this Example it will be

$$POq - Doq : PD :: POq : PA$$
$$112 : 20 :: 256 : 45.7143.$$

Now, if from P A you ſubtract P D, you have the Altitude of the Parabolic Conoid, whoſe Baſe is *c* D *o*. And ſince every Parabolic Conoid is equal to half a Cylinder of the ſame Baſe and Altitude, the Area of the Baſe O P O multiplied by half the Altitude P A, gives the Content of the Parabolic Conoid, whoſe Baſe is O P O, and the Altitude A P. And the Area of the Baſe *c* D *o*, multiplied by half the Altitude D A, gives the Content of the Parabolic Conoid whoſe Baſe is *c* D *o*, and Altitude D A. Therefore, if from the Content of the former you ſubtract the Content of the latter, you have the Content of the Fruſtum *c* D *o*, O P *o*.

Example. *c* D *o* is 24 Inches, the Area is 1.6042, this multiplied by half D A (*viz.*) 12.8571, gives 20.62 Gallons, the Content of the Parabolic Conoid D *c c* A *o o*, this ſubtracted from 65.18 (the Content of the Parabolic Conoid O *c* A *o o*) leaves 44.56 Gallons, the Content of the Fruſtum (*c* D *o*, O P O,) which was demanded. But the Content of any ſuch Fruſtum may be more readily found, thus,

To the Area of the greater Baſe, add the Area of the leſſer Baſe, the Sum multiplied by half the Altitude gives the Content.

Example. In the Fruſtum laſt mentioned,

The Diameter O P O, being 32 Inches, the Area is ———	2.8519
The Diameter *c* D *o*, being 24 Inches, the Area is ———	1.6042
Sum is ———	4.4561

This multiplied by half the Altitude (*viz.* 10) gives 44.561 Gallons, the Content, as before.

Now, Suppoſe the Fruſtum O B G O, be equal to the Fruſtum O *c* D *o* O, and the Baſe of O P O, common to both, this is what ſeveral Writers would have to repreſent a common Caſk, *viz.* a Butt, Hogſhead, Barrel, *&c.* But it is plain from the nature and conſtruction of the Figure, that no ſuch Caſk can be in the Form of theſe two Fruſtums abutting upon the common Baſe O P O. But if it were practicable to make a Caſk in this Form, if the Diameters, and the Length be as above mentioned, the Content will be 89,122 Gallons.

To find the Content upon every Inch of the Fruſtum. *c* D *o*, O P O.

From the Square of (O P O) the Diameter of the greater Baſe, ſubtract the Square of (*c* D *o*) the Diameter of the leſſer Baſe; and divide this Difference by D P the Depth, the Quotient is the common Difference of the Squares of the ſeveral Diameters, taken one the Middle of every Inch of D P. But for the Diameter in the Middle of the firſt Inch next the leſſer Baſe, take half this common Difference, and add it to the Square of *c* D *o*, the Sum is the Square of the Diameter in the Middle of that Inch; to this add the common Difference, the Sum will be the Square of the Diameter in the Middle of the ſecond Inch, and ſo of the reſt. But having got the Squares of the Diameters in the Middle of the firſt, ſecond, and third Inch, find the Area's of thoſe Circles in Gallons, the Difference of theſe Area's will be equal.

EXAMPLE.

EXAMPLE.

O P O = 32 Square = 1024
c D *o* = 24 Square = 576

Difference = 448

448 divided by D P 20, is 22.4 the common Difference.

Square of *c* D *o* = 567
half com. Differ. = 11.2

Square of the Diameter in the middle of the firſt Inch = 587.2 578.2
Common Difference = 22.4

Square of the Diameter in the middle of the ſecond Inch = 609.6
Common Difference = 22.4

Square of the third Diameter = 632.0

The Square of theſe Diameters thus found, multiplied ſeverally by .002785, give the Area or Content of the firſt, ſecond and third Inch.

	Area's	Diff.
Viz.	1.6354	
	1.6978	.0624
	1.7602	.0624

And ſo in the reſt, as in the following Table, in the aſt Column of which are the common Difference of he Area's. In the third Column, you have the Area or Content of every Inch. The firſt Number in the ſecond Column, *viz.* 44.5640 is the Sum of all the Area's in the third Column, that is the Content of the whole Tun: From which the ſeveral Area's continually ſubtracted, leaves the Number of Gallons contained in the Tun at any Depth. Thus againſt 10 Inches deep, you have 25.402 Gall. in the Tun, and ſo for any other Depth.

Inches of the Tun's Depth.	Content at any Depth.	Area's	Com. Diff.
20	44.5640	1.6354	.0624
19	42.9286	1.6978	.0624
18	41.2308	1.7602	.0624
17	39.4706	1.8226	.0624
16	37.6480	1.8850	.0624
15	35.7630	1.9474	.0624
14	33.8156	2.0098	.0624
13	31.8058	2.0722	.0624
12	29.7336	2.1346	.0624
11	27.5990	2.1970	.0624
10	25.4020	2.2594	.0624
9	23.1426	2.3218	.0624
8	20.8208	2.3842	.0624
7	18.4366	2.4466	.0624
6	15.9900	2.5090	.0624
5	13.4810	2.5714	.0624
4	10.9096	2.6338	.0624
3	8.2758	2.6962	.0624
2	5.5796	2.7586	.0624
1	2.8210	2.8210	.0624

Of a Parabolic Spindle.

IN Fig. 5. RH is 40, $Rs = bE$ 20, AR 16, Rb 12, and bA 4. And

$$AR : RHq :: Ab : bEq.$$
$$16 : 1600 :: 4 : 400.$$

Therefore (the other Side being made like to this) the Curve Line GCAEH is a Parabola. Now if this Parabola be turned about upon the Line GH, it will generate the Solid GAHBG, which is called a Parabolic Spindle. And every Parabolic Spindle is to its circumſcribing Cylinder, as 8 to 15. (*Vide* Dr. *Wallis*'s *Mechanica*, Part II. Chap. v. Prob. 12.)

PROB. IV. Fig. 5. Plate IV.

To deſcribe a Parabolic Spindle, the middle Fruſtum whereof ſhall repreſent a Caſk whoſe Diameter at the Bung AB, *is* 32 *Inches, the Diameters, at the Heads* DC, *and* EF, *are each* 24 *Inches, and the Length* P*s* 40 *Inches. And to find the Content of the middle Fruſtum thereof, repreſenting the Caſk propoſed.*

1. DRAW the ſtraight Line zz, and at the Right-angles to it, draw the Lines AB, CD, and EF, ſo as RP and RS, be each = 20. Make AR = 16.

= 16. CP, *b* R and E *s*, each = 12, ſo ſhall A *b*, be = 4. And becauſe it is a Parabola, it will be

$$Ab : AR :: bE_q : RH_q.$$
$$4 : 16 :: 400 : 1600$$

The Square-Root of 1600, is 40, = RH, or RG.

2. To find a Diameter in the Middle of every Inch of RG, or RH. Through the Point A, draw IK Parallel, and equal to the Line HG, let IA and GR, be each divided into half Inches, which here will be 80, the Square of this is 6400, by which divide AR 16, the Quotient will be .0025, this multiplied by the Squares of all the odd Numbers, *viz.* 1, 3, 5, 7, *&c.* to 70, the ſeveral Products give the Length of the Line *n e*, which taken from the Line AR, leaves the Lines *e o*, the Semidiameters of the Parabolic Spindle (one in the Middle of every Inch of GR.) But to contract this Work, we need but the three firſt Diameters, the reſt are obtained by ſubtracting the common Difference; as will appear by the following Operation.

$$\begin{matrix}.0025\\ \text{Multipli-}\\ \text{ed by}\end{matrix}\left\{\begin{matrix}1\\9\\25\end{matrix}\right\} \text{is} \left\{\begin{matrix}.0025\\.0225\\.0625\end{matrix}\right\} \text{Each of}$$

theſe taken from AR 16, the three Remainders will

$$\text{be}\left\{\begin{matrix}15.9975\\15.9775\\15.9375\end{matrix}\right\}\begin{matrix}\text{theſe mul-}\\ \text{tiplied by}\\ \text{2 give —}\end{matrix}\left\{\begin{matrix}31.995\\31.955\\31.875\end{matrix}\right\}\text{the}$$

three Diameters, one in the Middle of every Inch next to AB; to find the common Difference, take the ſecond Diameter from the firſt, there reſts .040; take the third from the ſecond, the Remainder is .080; take .040 from .080, there reſts .040, the Second Difference, which is the ſame throughout; and by this common Difference the reſt of the Diameters are thus found. Take the firſt Difference (*viz.* .040) from the firſt Diameter, the Remainder is the ſecond Diameter; add the ſecond Difference to the firſt Difference, and ſubtract their Sum from the ſecond Diameter, the Remainder will be the third Diameter; and ſo of the reſt, as in the following Table; in the ſecond Column of which you have the ſecond Difference (which is the ſame throughout) in the third Column are the firſt Differences, in the fourth Column you have the true Diameters of the Spindle, one in the Middle of every Inch of R P, and in the laſt Column are the Area's to theſe Diameters, that is, the Content of ſo many circular Plains of one Inch thick, which added together make 48.249 Ale-Gallons, the Content of the Part B D C A, this doubled is 96.498, the Content of the middle Fruſtum C A E F B D, repreſenting the Caſk propoſed.

Note, The Area's in this Table were found by multiplying the Squares of each Diameter by .002785. But this Work may be much ſhortened; for if the Squares of the ſeveral Diameters in the fourth Column be added together, and their Sum multiplied by .002785, the Product will be 48.349 = the Sum of the Area's in the laſt Column.

A TABLE ſhewing the Content of half the middle Fruſtum of the PARABOLIC SPINDLE, CAEFBD. Fig. 5. Plate IV.

	Second Differ.	Firſt Diffe-rence.	Diameter in the Middle of every Inch of R P.	Area in Gallons, or the Content of every Inch from R to P.
1			31.995	2.851
2	40	40	31.955	2.844
3	40	80	31.875	2.830
4	40	120	31.755	2.808
5	40	160	31.595	2.780
6	40	200	31.395	2.745
7	40	240	31.155	2.703
8	40	280	30.875	2.655
9	40	320	30.555	2.600
10	40	360	30.195	2.539
11	40	400	29.795	2.473
12	40	440	29.355	2.400
13	40	480	28.875	2.322
14	40	520	28.355	2.239
15	40	560	27.705	2.152
16	40	600	27.195	2.060
17	40	640	26 555	1 964
18	40	680	25.875	1.865
19	40	720	25.155	1.762
20	40	760	24.395	1 657

Sum——48.249
Content of the Caſk——96.498

Note, If the Part ABDC were a Tun ſtanding upon its leſſer Baſe DC, and from the whole Content thereof, *viz.* 48.249, you ſubtract the firſt Area, *viz.* 2.851, the Remainder 45.398, will be the Content at 19 Inches deep; and if from this you take the ſecond Area, you will have 42.554, the Content at 18 Inches deep, and proceeding with the reſt in the ſame manner, you will have a Table ſhewing the Content at any Depth.

Of an Hyperbola.

IN Fig 1. the Cone SVB, is cut by a Plain HA, which alſo cuts one Side of the Triangle produced in E, therefore the crooked Section O*oo* A *oo*O, is an Hyperbola. In which EA is the tranſverſe Diameter, and AH the intercepted Diameter. Now if the Cone be cut by another plain Parallel to the Baſe, this Section will be a Circle whoſe Diameter is SHB: And $SH \times HB = HOq$, therefore HO is an Ordinate rightly applied, and if HO be an Ordinate rightly applied, it will be

$$AH \times EH : Ab \times Eb :: HOq : boq.$$

PROB. V. Fig. 4.

To find the Content of an Hyperbolic Conoid, or the Content of any Part thereof in Ale-Gallons.

IF the Semi-Hyperbola A *s s* S D be turned round about upon the Line A D, it will deſcribe the Hyperbolic Conoid S *s* A *o* O. Now, if we put the Tranſverſe Diameter $= t$, the Intercepted Diameter $= d$, the Magnitude of the Hyperbolic Conoid $=$ A, and the Magnitude of a Cylinder of the ſame Baſe and Altitude $=$ B, it will be

$$6t + 6d : 3t + 2d :: B : A.$$

That is, (1.) To 6 times the Tranſverſe Diameter A E, add 6 times the Intercepted Diameter A D, the Sum is your Diviſor.

(2.) To 3 times the Tranſverſe Diameter, add twice the Intercepted Diameter, by this Sum multiply the Content of the Cylinder N V O S, the Product is your Dividend; which divided by the Diviſor above found the Quotient will be the Content of the Hyperbolic Conoid, whoſe Baſe S D O is the Baſe of the Cylinder N V O S, and the Altitude A D equal to S N the Altitude of the Cylinder; and this will hold whereſoever the Baſe be taken: For ſuppoſe the firſt (*s d o*,) next the Vertex A, be taken for the Baſe of the Hyperbolic Conoid, *s* A *o*, (and of the Cylinder *s n n o*,) the Content of the Hyperbolic Conoid, may be found as above, and the Content of any Fruſtum, S D O *o* D *s* may be likewiſe found; for if the little Hyperbolic

Conoid *ſ* A *o d*, be taken from the Hyberbolic Conoid S *ſſ* A *oo* O, there remains the Fruſtum *ſ* D O *o d ſ*, and thus the Content of the whole, and alſo the Content of any Fruſtum, and conſequently of every Inch of the Hyperbolic Conoid may be readily found, as will appear in the following Example.

EXAMPLE. Fig. 4.

Let A E, the Tranſverſe Diameter, and A D, the Intercepted Diameter, be each 32 Inches, and S D O the Diameter of the Baſe 45.255 Inches, to find the Content of the Hyperbolic Conoid in Ale-Gallons.

1. Six times A E is 192, and 6 times A D is 192, the Sum of theſe is 384, for the firſt Term of the Proportion.

2. Three times A E is 96, and twice A D is 64, the Sum of theſe is 160, for the ſecond Term.

3. The Diameter of the Baſe S D O is 45.255 (equal the Square-Root of 2048) and the Area of this Circle is 5.704 Gallons; this multiplied by 32 (equal the Altitude S N) gives 182 528 the Content of the Cylinder N S O V, and this is the third Term of the Proportion, which will ſtand thus;

384 : 160 : : 182 : 528.

Now multiply the third Term by the Second and divide that Product by the Firſt, the Quotient or fourth Term will be 76.053 Ale-Gallons, which is the Content of the Hyperbolic Conoid required.

And by the ſame Method the Content of the little Hyperbolic Conoid, whoſe Altitude *d* A, is 4 Inches, and the Diameter of its Baſe (*ſ d o*) 16.97 Inches, will be found to be 1.545 Gallons. This taken from the Content of the Hyperbolic Conoid above found, the Remainder is 74.508 Ale-Gallons, the Content of the Fruſtum S D O *o d s*; and ſo for any other Fruſtum.

PROB. VI. Fig. 6.

To deſcribe an Hyberbola *in* Plano *Geometrically.*

DRAW a ſtraight Line R R; and from the middle Point W, let fall the Perpendicular W B, then divide W R into any Number of Parts, and draw the Lines *r o* G, *r o* H parallel to W B, then ſet one Point of your Compaſſes in the Line W B, ſuppoſed at A, and extend the other to *r*, where keep it faſt whilſt the firſt Point, with the ſame Extent, is removed from A, to Line *r* G, it will fall upon the Point *o*, and ſo on the other Side. I ſay, the Points *o o o*, are in the *Hyperbola*, and by a competent Number of ſuch Points thus found, the *Hyperbola* may be drawn. WA is half the Tranſverſe Diameter, A B the Intercepted Diameter, and B *o* an Ordinate rightly applied. (*Vid.* Dr. *Wallis*'s *Mechanica*, Part II. Page 553.)

Note, If the *Hyperbola o o* A *o o*, be turned about upon the Line Z Z, it will deſcribe the Solid *o o* A *o o o* F D E *o*, which may be called an Hyperbolic Spindle.

PROB.

PROB. VII. Fig. 6.

To deſcribe an Hyperbolic Spindle, the middle Fruſtum of which ſhall repreſent a Caſk whoſe Diameter at the Bung A D, *is* 22 *Inches, the Diameters at the Heads, o* E, *and o* F, *are each* 34 *Inches, and the Length* G H, 40 *Inches, and to find the Content of this Fruſtum in Ale-Gallons.*

DRAW the right Line Z Z, and at right Angles thereto, the Line W D, make B G and B H, each equal to 20 (the Semi-length of the Caſk propoſed) through the Points G and H, draw the Lines *r* E, and *r* F parallel to W D, make G *o* and H *o*, each equal to 12, and B A 16, then make B *b* equal to G *o*, ſo ſhall A *b* be 4. I ſay, the Square *b o*, (400) divided by A *b*, (4) is equal to 100, from which ſubtract 4, the Remainder (96) is the Tranſverſe Diameter of this Hyperbola; make AW equal to 48, (the half of 96) and through the Point W, draw the Line R R parallel to Z Z, and let the Line *r o* be parallel to W D; I ſay, the Square of W A, more the Square of W *r*, is equal to the Square of *r o*, of which the Square-Root is the Length of the Line *r o*. Now the Line *r o*, taken from the Line W B, the Remainder is the Line *o g*, which doubled give *o e*, a Diameter in the Spindle, and this will hold whereſoever *r* be taken, and ſo innumerable Diameters *o e*, may be fonnd on both Sides A D, to the Ends of the Spindle. But for our preſent Purpoſe, it will be ſufficient to find a Diameter in the Middle of every Inch of the Part A D E *o*, as ſuppoſe the firſt Diameter *o g e*, be half an Inch from A D, the firſt W *r* is .5, the Square of this (*viz.* .25) added to the Square of W A (2304) is equal to the Square of *r o* (2304.25) the ſecond W *r*, will be .25, the third 2.5, *&c.* the Square of which you have in the firſt Column of the following Table. In the ſecond Column are the Sums of the Squares of W A and W *r*, that is, the Squares of *r o*. In the third Column are the ſquare Roots of the Numbers in the ſecond Column, that is, the Length of the Lines *r o*. Now each of theſe Numbers taken from W B, 64, leaves the Lines *o g*, which doubled are the Diameters *o c*, as in the fourth Column. In the laſt Column are the Area's of theſe Circles in Ale-Gallons, that is, the Content of ſo many Circular Plains of one Inch thick, the Sum of theſe is 48.1 (equal the Part A D E *o*) which doubled is 96.2, equal to the middle Fruſtum *o* E D F *o* A. The Content here found is not Mathematically True, but near enough for any ordinary Practice; and by the Method here laid down, the Content may be found ſo near the Truth, that the Difference ſhall be leſs than any Quantity aſſignable.

Now, Let the Part D E *o* A, repreſent a Tun, and ſuppoſe it to ſtand upon the leſſer Baſe E *o*. If from the whole Content thereof, *viz.* 48.1, you ſubtract the firſt Area, *viz.* 2.851, the Remainder 45.249, will be the Content at 19 Inches deep; and if from this you ſubtract the ſecond Area, *viz.* 2.843, you will have 42.406, the Content at 18 Inches deep; and proceeding with the reſt after the ſame Manner, you will have a Table ſhewing the Content of this Tun at any Depth.

A Table ſhewing the Content of every Inch of half the middle Fruſtum of the Hyperbolic Spindle E *o* A *o* F D (Fig. VI.) in Ale-Gallons.

Squares of W *r*.	Sum of the Squares of W A, and W *r*.	Square Roots of the Square of (*r o*) or the length of the Lines *r o*.	Diameter *o e*.	Area's of the Circle in Ale-Gallons.
.25	2304.25	48.003	31.995	2.851
2.25	2306.25	48.023	31.953	2 843
6 25	2310.25	48.065	31.870	2 829
12.25	2316.25	48.127	31.745	2.807
20.25	2324.25	48.210	31.579	2.777
30.25	2334.25	48.314	31.372	2.741
42.25	2346.25	48.438	31.124	2.698
56.25	2360.25	48.582	30.835	2 648
72.25	2376.25	48 747	30.506	2.592
90.25	2394.25	48.931	30.138	2.520
110.25	2414.25	49.135	29.730	2.462
132.25	2436.25	49.358	29 283	2 388
156.25	2460.25	49.601	28.798	2.310
182.25	2486.25	49.862	28.275	2 227
210.25	2514.25	50.142	27 715	2.139
240.25	2544.25	50.441	27.119	2.048
272.25	2576.25	50.757	26.486	1.954
306.25	2610.25	51.090	25.819	1.857
342.25	2546.25	51.442	25.117	1.753
380.25	2684.25	51.809	24 380	1.656

Sum is ———————— 48.100

This doubled is ———————— 96.200

The Content of the Fruſtum E *o* A *o* E D.

Of an Ellipſis.

IF the Cone S *e* B *d* V (Fig. 2.) be cut by a plain (A *a*) through both the Sides of the Triangle, the crooked Section *a o o* A *o o* will be an Ellipſis, in which A *a*, is the Tranſverſe Diameter, or Axis, and A D, a Diameter intercepted.

Let the Cone be cut by another Plain (S B) parallel to the Baſe S *e* B *d*, it will be $SD \times BD = Deq$, and this will hold whereſoever D *b e* is taken, and give the Lines D *o*, which are called Ordinates rightly applied, and the Lines *o* D *o* (cutting the Line *a* A, at Right Angles) are called Conjugate Diameters of the Ellipſis.

PROB. VII. Fig. 7.

Given the Axis of the Ellipſis, and its greateſt Conjugate Diameter (cutting the Axis at Right Angles) to deſcribe the Ellipſis.

DRAW a ſtraight Line, and make A *c* equal to the Axis given, through the middle Point B, and at Right Angles to A *c*, draw the Line F L, and from the Point B, ſet off half the Conjugate Diameter to *o* and G, draw the Lines *d b*, parallel to F L. Join B D, and B *d*, upon the Centre B, and with the Interval B A, deſcribe the Circle F A L C. I ſay, $BDq - BEq = EDq$. And BF : B : : E D : E *o*. This will hold Whereſoever E be taken, and give the Points *o o*, (as many as you pleaſe on both Sides) through which the Ellipſis may be drawn.

Of a Spheroid. Fig. 7.

IF the Semi-Ellipſis A *o c*, be turned about upon the Line A *c*, it will generate a Solid A G *c o*, which is called a Spheroid.

Note, 1. The Content of every Spheroid is equal to two Thirds of the Content of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheroid; and its Altitude equal to the Spheroid's Axis. Therefore the Area of the greateſt Circle of a Spheroid, multiplied by two Thirds of its Axis gives the Content.

2. If the Spheroid be cut by a Plain Z Z, parallel to the Line *o* G, and through the middle of B *c*, the Content of the Fruſtum Z *c* Z, to the Content of a Cylinder, whoſe Baſe is Z Z, and Altitude V *c*, is as 5 to 9. Therefore, the Area of the Baſe Z Z, multiplied by five Ninths of V *c*, gives the Content of the Fruſtum Z *c* Z.

3. If the Spheroid be cut as above, the upper Fruſtum Z *o* A H G Z, is equal to the Cylinder, whoſe Baſe is Z Z, and its Altitude V A. Therefore the Area of the Baſe Z Z, multiplied by the Altitude V A, gives the Content of the Fruſtum Z *o* A G Z. Theſe three Propoſitions do alſo hold in a Sphere. *Vid. Dr.* Wallis's *Treatiſe of Algebra, Page* 312.

PROB.

PROB. VIII. Fig. 7.

To deſcribe a Spheroid, the middle Fruſtum of which ſhall repreſent a Caſk, whoſe Bung-Diameter o G, *is* 32 *Inches, the Diameters at the Heads* o H *and* o K, *are each* 24 *Inches, and the Length* E E 40 *Inches. And to find the Content of ſuch Caſk upon every Inch of the Depth* E E.

FROM the Square of (*o* G) the Bung-Diameter, ſubtract the Square of (*o* H) the Head-Diameter. Then,

As that Difference,
  Is to the Square O G,
So is the Square of B E,
  To the Square of B A.

The Square Root of B A, is the Semi-Axis of the Spheroid, with which, as Radius deſcribe the Circle F A L *c*, then find the Points *o o*, as in the laſt Problem, and ſo the Diameters *o* H, *o* H, as many as you pleaſe, one in the middle of every Inch of E B, will be ſufficient: For the Area's of theſe Circles in Gallons, being added together, give the Content of the Part *o* G H *o*, and this doubled is the Content of the Caſk *o* H K *o*.

But for our preſent Purpoſe, it will be ſufficient to find the Square of the Diameter in the Middle of each of the three firſt Inches, from B towards K, the Squares of the reſt, and alſo the Area's of their Circles being eaſily obtained by their common Difference. Thus, From the Square (*o* G) the Bung-Diameter, ſubtract the Square of (*o* H) the Head-Diameter, and divide the Square-Root of the Difference by (B E) the Semi-Length of the Cask. Then multiply this Quotient by .5, 1.5 and 2.5, and call the Products S.

Now, if from the Square of (*o* G) the Bung-Diameter, you ſubtract the Squares of S, the Remainders will be the Squares of the reſpective Diameters ſought.

o G = 32 Square is = 1024
o R = 24 Square is = 576
Difference = 448

The Square Root of 448, is 21.16601, this divided by (B E) 20, gives 1.0583, which multiplied

by	.5	is	.52915	= to S	.28
	1.5		1.58645	which	2 52
	2.5		2.64575	ſquared is	7.00

Now the Square of *o* G, *viz.* 1024.

Leſs the	.28	is	1023.72
Square	2 52		1021.48
of S, *viz.*	7.00		1017.00

which are the Squares of the three firſt Diameters one in the Middle of every Inch from *o* G, towards *o* K, or *o* H; and theſe multiplied ſeverally be .0427851, gives the Area's of the reſpective Circles in Ale-Gallons.

$$viz.\left\{\begin{array}{l}2.85116\\2.84492\\2.83244\end{array}\right\}\begin{array}{l}.00624\\.01248\end{array}\quad .00624$$

Now if the ſecond Inch be ſubtracted from the firſt, the Difference is .00624, and the third Inch ſubtracted from the ſecond, the Difference is .01248, then, from .01248, ſubtract .00624, the Difference will be .00624, which is the ſame throughout as in the following Table. In which obſerve, that the firſt Differences are made by a continual Addition of the ſecond Difference, and the Area's in the ſecond Column are obtained by ſubtracting the firſt of the firſt Differences from the firſt Area, and ſo on. Theſe Area's added together are 48.724 Gallons equal to the Content of the Part *o* G K *o*, which doubled is 97·448 the Content of the Cask *o* H K *o*, which was required.

A TABLE ſhewing the Content of every Inch of half the middle Fruſtum of the Spheroid, *o o* K G. Fig. 7.

	Content of every Inch of the Part *o* G K *o*	1. Differ.	2. Differ.
1	2.85116		
2	2.84492	.00624	.00624
3	2.83244	.01248	.00624
4	2.81372	.01872	.00624
5	2.78876	.02496	.00624
6	2.75756	.03120	.00624
7	2.72012	.03744	.00624
8	2.67644	.04368	.00624
9	2.62652	.04992	.00624
10	2.57036	.05616	.00624
11	2.50796	.06240	.00624
12	2.43932	.06864	.00624
13	2.36444	.07488	.00624
14	2.28332	.08112	.00624
15	2.19526	.08736	.00624
16	2.10236	.09360	.00624
17	2.00252	.09984	.00624
18	1.89644	.10608	.00624
19	1.78412	.11232	.00624
20	1.66556	11856	.00624

Total = 48.724

The Double is 97.448 = Content of the Cask.

Of

Of a Circular Spindle.

IF the Arch of a Circle Z A Z (Fig. 8.) be turned about upon its Cord Z B Z, it will deſcribe a Solid Z *o* A *o*, Z F *c* E, and this Solid, I call a Circular Spindle.

PROB. IX. Fig. 8.

To deſcribe a Circular Spindle, the Middle Fruſtums of which ſhall repreſent a Cask, whoſe Bung-Diameter A C, *is* 32 *Inches, the Diameters at the Heads o* E, *and o* F *are each* 24 *Inches, and the Length* G H, 40 *Inches. And to find the Content of ſuch Cask in Ale-Gallons.*

DIVIDE the Square of the Semi-Length of the Cask, by half the Difference between the Bung and Head-Diameters, and to the Quotient add the ſaid half Difference; the half of this Sum is equal to the Radius A D; with which (upon D as a Centre) deſcribe the Arch L A M, draw the Line L M, and make A D perpendicular thereto. Make A B equal to 16, the Semi-Diameter at the Bung. Draw the Line Z Z parallel to L M, and make B *c*, equal to B A. Through the Points Z *a* Z, deſcribe the Arch Z E *c* F Z, ſet off the Semi-Length of the Cask from B, to G and H, through which Points, draw Lines parallel to A D. So ſhall *o* E and *o* F, be the Head-Diameters of the Cask, and each 24.

To find a Diameter in the middle of every Inch from A *c*, towards *o* E. Put S = *b o*. It is manifeſt, that the Square of *o* D, leſs the Square of S, is equal to the Square of *o g*. And if from the Square-Root of *o g*, you take *o g*, the Remainder G *o*, will be the Semi Diameter; which doubled gives the Diameter *o* E. And if you put the firſt S = .5, the ſecond = 1.5, the third = 2.5. *&c.* and proceed as here directed, we ſhall have a Diameter in the Middle of every Inch, as in the ſecond Column of the following Table, againſt which, in the third Column, are the Area's in Ale-Gallons, that is, the Content of ſo many Circular Plains of one Inch thick, the Sum of theſe is 48.3938, the Content of the Part *c* E *o* A; which doubled is 96.7876, the Content of the Cask required.

A TABLE ſhewing the Content of half the middle Fruſtum of the Circular Spindle *o* E *c* F *o* A. Fig. 8.

	Diam. in the Middle of every Inch of B. G.	Area in Ale-Gallons.
1	31.9952	2.8510
2	31.9567	2.8442
3	31.8797	2.8305
4	31.7642	2.8090
5	31.6098	2.7828
6	31.4166	2 7491
7	31.1843	2.7084
8	30.9126	2.6614
9	30.6012	2.6309
10	30.2497	2.5464
11	29.8577	2.4828
12	29.4248	2.4113
13	28.9505	2.3342
14	28.4340	2 2517
15	27.8749	2.1640
16	27.2723	2.0615
17	26.6255	1.9744
18	25.9336	1.8733
19	25.1957	1 7680
20	24 4016	1.6589

Sum = 48.3938
Doubled is = 96.7876 = Content of the Caſk.

From what hath been ſaid, it appears, that the middle Fruſtum of each of the four Solids laſt mentioned, repreſent a cloſe Caſk, (*viz.* a Butt, Pipe, Hogſhead, *&c.*) And if a Caſk whoſe Diameters and Length are as above ſuppoſed.

viz.	Bung	Diameter	32	Inches.
	Head		24	
	Length	———	40	

I ſay,

	Ale-Gall.
I ſay, if ſuch Caſk be the middle Fruſtum of a Spheroid (as *o* H K *o*. Fig. 7.) the Content (by Prob. VIII.) will be ——	97.448
If the middle Fruſtum of a Circular Spindle, (as *o* E F *o*, Fig. 8.) the Content (by Prob. IX.) will be —— —— ——	96.787
If the middle Fruſtum of the Parabolic Spindle, (as D C E F, Fig. 5.) the Content by Prob. IV. will be ——	96.498
If the middle Fruſtum of a Hyperbolic Spindle (as *o* E F *o*, Fig. 6.) the Content by Prob. VI. will be ——	95.200
And if (as ſome Writers ſuppoſe) the ſaid Caſk be the middle Fruſtum of two Parabolic Conoids, abutting upon one common Baſe (as *o* D *o* O B G O, Fig. 3.) the Content by Prob. 3. will be ——	89.122
If the middle Fruſtum of two Cones abutting upon one common Baſe, the Content will be —— ——	87.934

But the two laſt rarely, if at all, occur in the Practice of Gauging.

And though in ſeveral Books on this Subject we meet with the Terms Fruſtum of a Spheroid, Conoid, Parabolic Conoid, Parabolic Spindle, *&c.* the Writers of ſuch Tracts have not, that I can find, told us what they mean by theſe Terms, nor will the Rules they have given find the Content of ſome of theſe Solids tolerably near the Truth.

For Inſtance,

If the Caſk above-mentioned be the middle Fruſtum of a Parabolic Spindle, the Content (as we have ſhewn above) is 96.498, Ale-Gallons. Whereas Mr. *Jordain*, in his Duodecimal Arithmetic, makes it but 93.877.

Mr. *Dary* at the End of his Miſcellanies, makes the Content but 90.948.

And Mr. *Hunt*, in his Practical Gauging Epitomized, makes the Content of this Cask but 87.930, which is above 8 Gallons leſs than the true Content.

If what has been ſaid in this Appendix ſhall give any Light to the Practical Gauger, in conceiving the Nature and Properties of the Solids here mentioned, it is as much as was at firſt deſigned.

For although the middle Fruſtum of each of the four Solids deſcribed in Prob. IV, VI, VIII, and IX. very properly repreſent a Cask, and the Contents there found be ſufficiently Exact; yet the Contents of three of them differ ſo little one from another, that if every Cask were in the Form of one of thoſe three Solids it would be needleſs to make any Diſtinction, becauſe upon ſight of a Cask, one could not tell to which of thoſe Solids it agreed.

And there being very few, if any, cloſe Casks of ſuch Diameters and Lengths as we have frequently mentioned, which contain more than the middle Fruſtum of a Spheroid (this having the greateſt Curvature between the Head and Bung;) but very many Casks (of ſuch Diameters and Length) which will not contain ſo much as the middle Fruſtum of a Parabolic or Hyperbolic Spindle, (their Staves between the Head and Bung, being nearer a ſtraight Line than either of theſe Fruſtums) I muſt refer the Practical Gauger to the Rules laid down in the four firſt Problems of the Ninth Section of the foregoing Tract, by which the Content of any Cask may be readily found (either by the Pen or Inſtrument) ſufficiently near the Truth.

FINIS.

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Fig. I.

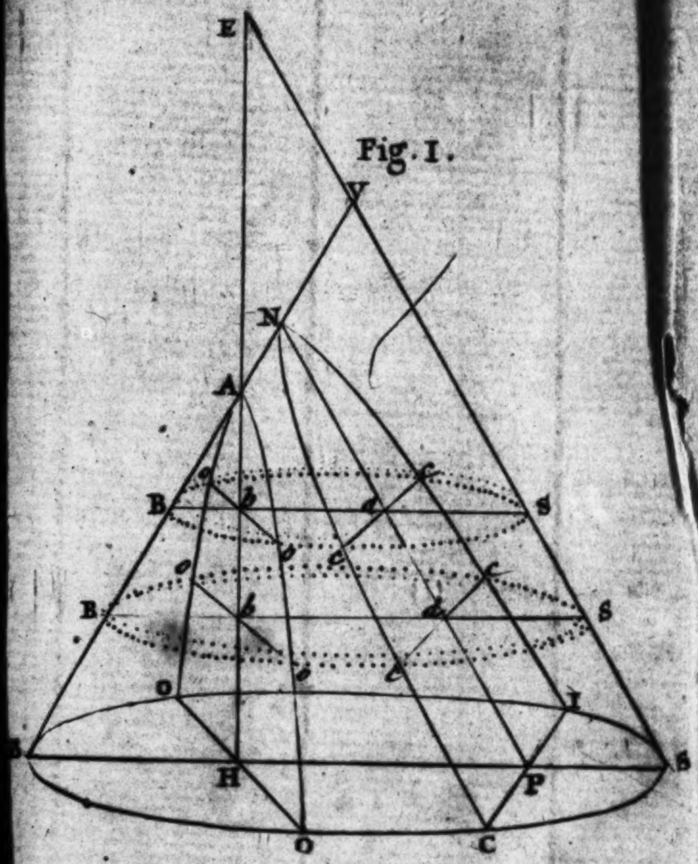


Fig. II.

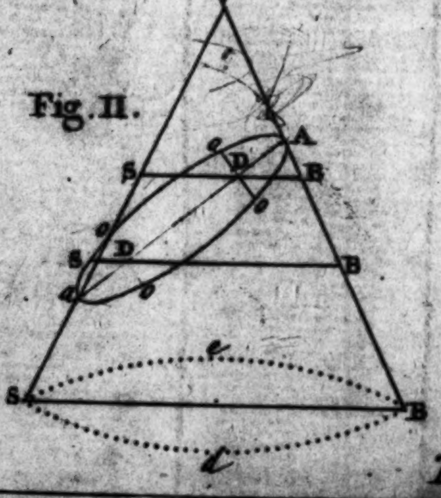
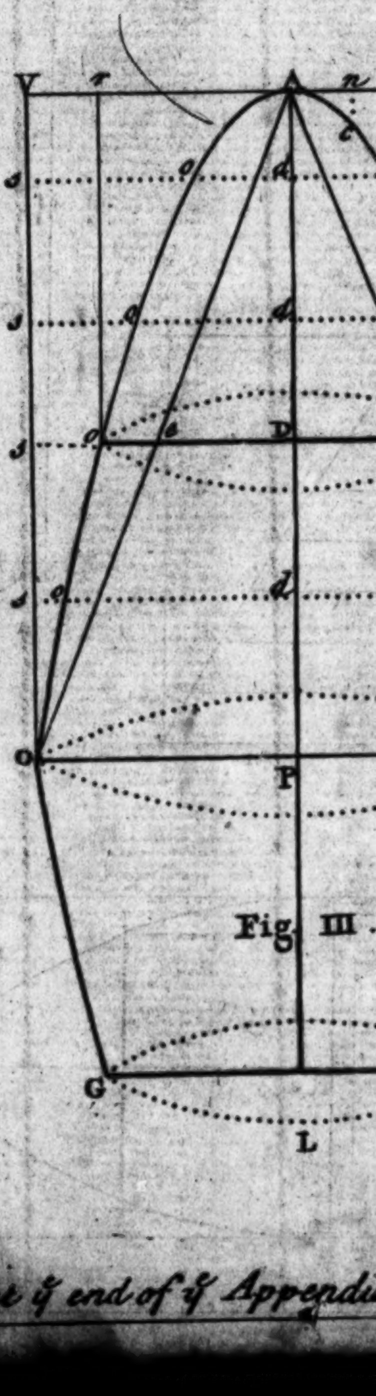


Fig. III.



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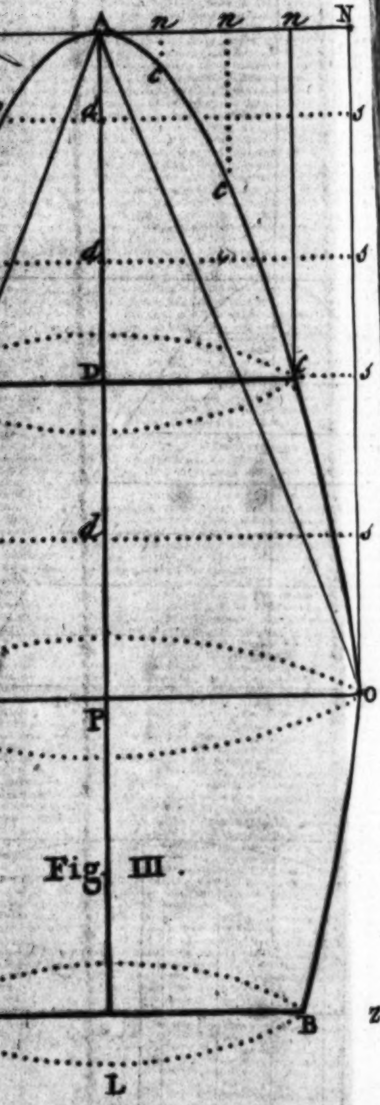


Fig. III.

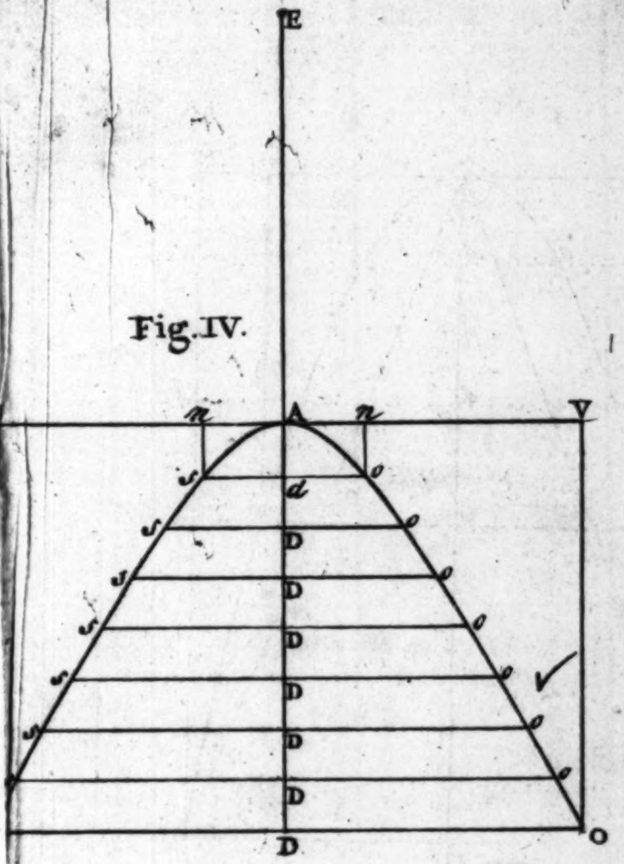


Fig. IV.

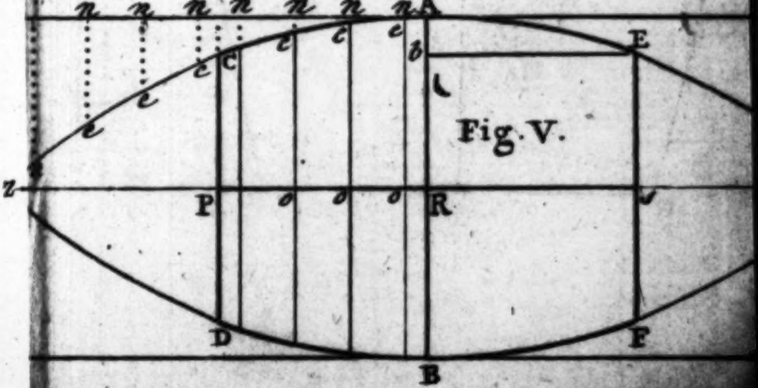


Fig. V.

f Appendix

Fig. IV.

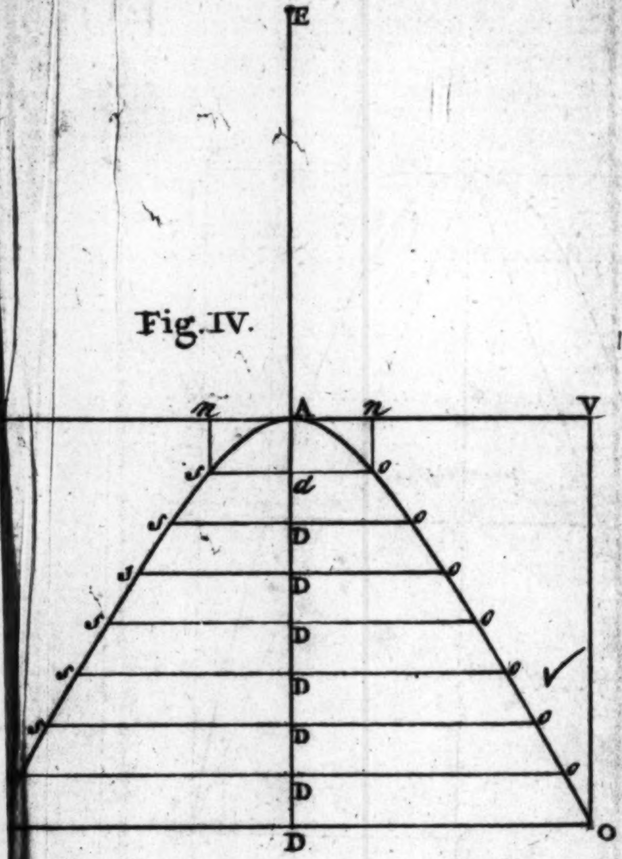


Fig. VI.

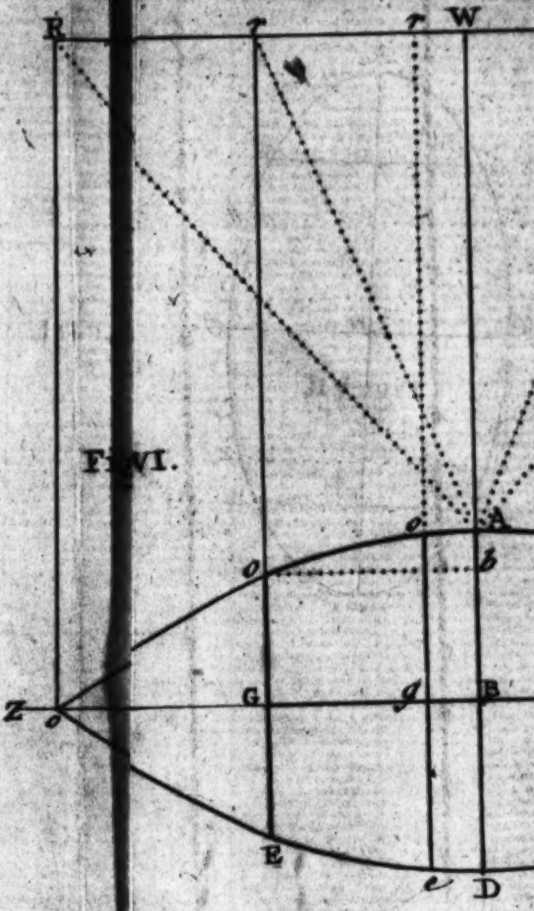


Fig. V.

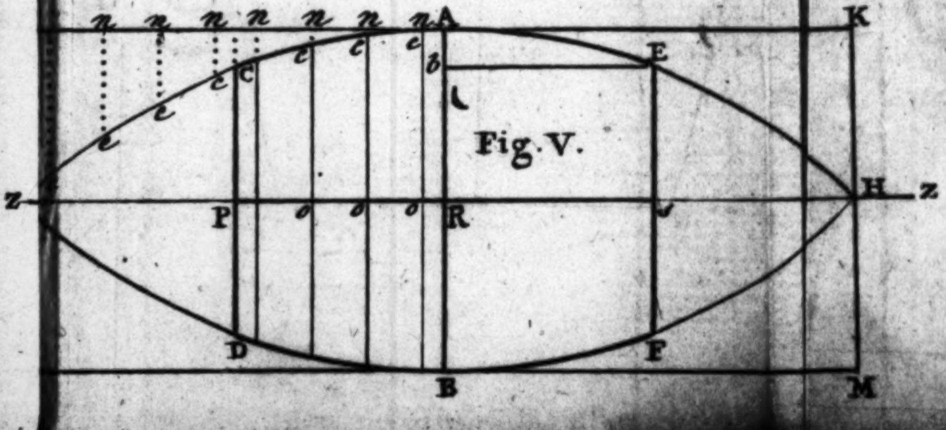
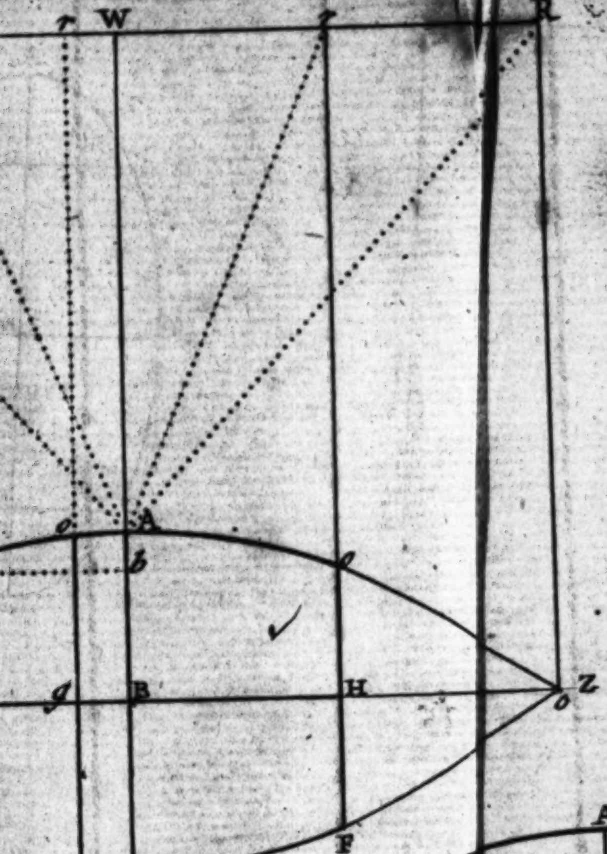


Plate. IV.



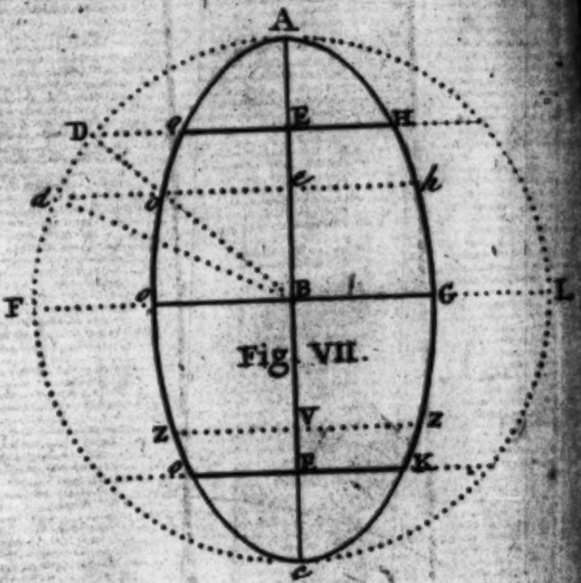


Fig. VII.

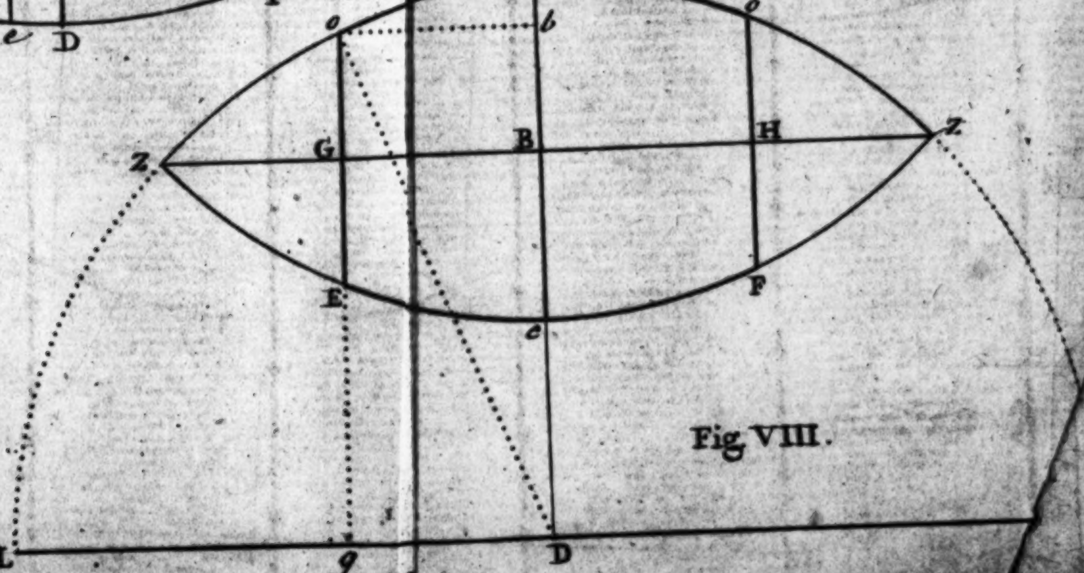


Fig. VIII.